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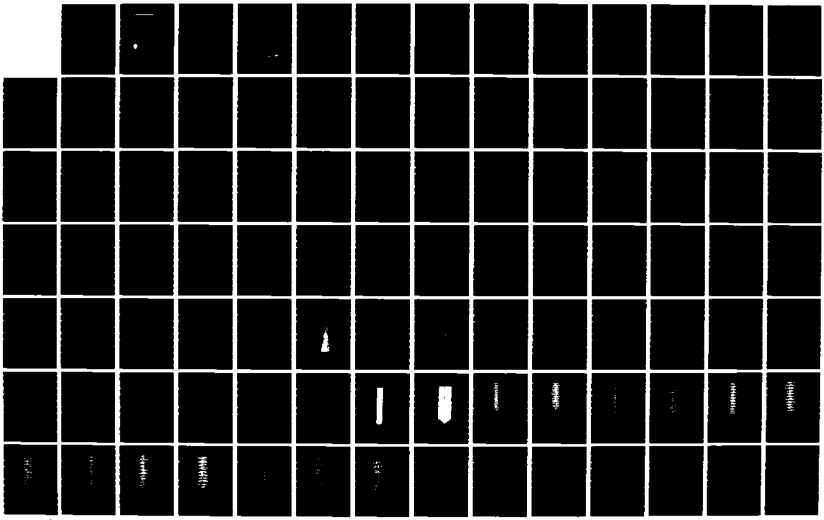

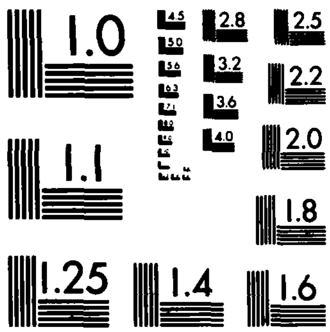

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A THEORETICAL ANALYSIS OF NONLINEAR EFFECTS ON THE FLUTTER AND DIVERGENCE OF A TUBE CONVEYING FLUID

ENOCH CH'NG, '78

AMS Report No. 1343

Department of
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ABSTRACT

A theoretical model capable of describing the flutter and divergence of a tube conveying high velocity fluid is developed in this study. The nonlinear effect due to tension induced by bending is considered along with other linear forces. The partial differential equations of motion are reduced to a set of ordinary differential equations by Galerkin's method and the time histories of the tube motion are evaluated numerically. Numerical results for the thresholds of flutter instability and the mode shapes of a cantilevered tube are in good agreement with those obtained theoretically and experimentally by other investigators. (1)

A static or divergence instability occurs at a certain critical fluid velocity if the tube is simply supported at both ends. At higher fluid velocity, the tube becomes dynamically unstable. The limit cycle oscillations of the cantilever tube and the characteristics of the simply supported tube motion predicted by the present nonlinear analysis remain to be verified quantitatively by experiment. Plans have been made to set up an appropriate experiment.

ACKNOWLEDGEMENTS

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NOMENCLATURE

a_{jk}	mode shape integral given by Eq. 41
a	cross sectional area of tube
A	notation for matrix
b_{jk}	mode shape integral given by Eq. 42
c_{hi}	mode shape integral given by Eq. A-15
ds	differential length
D	damping factor given in Eq. 9
E	Young's modulus
f_{damp}	damping force
g	acceleration due to gravity
I	moment of inertia of tube section
ℓ	length of tube
m	mass per unit length of tube
m_f	mass per unit length of fluid
$\vec{q}$	column vector of q_k
q_k	generalized coordinate

q^*	some constant q_k
r	radius of gyration
R	end reaction force
$\overline{R}$	radius of tube
$\overline{R}_1$	inside radius of tube
$\overline{R}_2$	outside radius of tube
t	time
t	thickness (Appendix III)
T	kinetic energy
T_o	tension
U	potential energy
V	velocity of fluid in tube
W	work
x	coordinate
y	displacement of tube
β	nondimensional velocity $= \sqrt{\frac{m_f \ell^2}{EI}}\, V = 3.52 \sqrt{\frac{\mu}{1-\mu}}\, \frac{V}{\omega_1 \ell}$
ε_x	strain (x-direction)
γ_k	k^{th} cantilever beam mode given in Eq. 34

γ_n	n^{th} simply supported beam mode given in Eq. 63
δ	variation
δ_{jk}	delta function
∇	operator, see Eq. 55
ζ_i	critical damping ratio of i^{th} mode without fluid
λ_k	mode shape constant of Eq. 35
μ	mass ratio = $\frac{m_f}{m_f + m}$
ν	viscous damping parameter = $7.04\ \zeta_1 \sqrt{1-\mu}$
ξ	nondimensional coordinate = x/ℓ
ϕ	nondimensional displacement = y/r
σ_k	mode shape constant of Eq. 35
σ_x	stress (x direction) in psi
τ	nondimensional time = $\sqrt{\frac{EI}{(m_f+m)\ell^4}}\, t$
θ_1, θ_2	angles of deflection
ω	frequency
$\bar{\omega}_i$	natural frequency of i^{th} mode without fluid
Ω	nondimensional frequency = $\sqrt{\frac{(m_f+m)\ell^4}{EI}}\, \omega = \frac{3.52}{\sqrt{1-\mu}} \frac{\omega}{\bar{\omega}_1}$

Subscripts and Superscripts

c	cantilevered
f	fluid
ss	simply supported at both ends
T	true
$(\dot{\ })$	$\frac{d}{d\tau}(\ )$
$(\ )'$	$\frac{d}{d\xi}(\ )$

PART I: INTRODUCTION

As early as 1950, the observation of bending vibration of a simply supported petroleum pipeline prompted scientific investigations (2). More recently, it was observed that a short pipeline vibrates in its second circumferential mode at fairly high frequency (300-800 Hz) above a certain critical flow velocity (see Figure 1). The vibrations may sometimes generate a shrill sound (3). If the pipeline is sufficiently long, this instability is then superposed on the flexural oscillatory instabilities (Figure 2). High performance liquid-propellant launch vehicles may require rapid transfer of large quantities of fluid from tanks to pumps through pipes with relatively thin walls. Thus, a study of the influence of fluid velocity on the static and dynamic stabilities of propellant lines (tubes) is necessary (4). Other applications occur in hydraulic lines and human lung airways (5).

The purpose of this paper is to analyze the observed motion of an elastic tube conveying fluid using a mathematical model based upon the following assumptions:

(a) The three basic classes of operative forces are inertial, elastic and aerodynamic forces coupled by the elastic deformations of the tube.

(b) The fluid being conveyed in the tube is inviscid, incompressible and non-heat conducting.

(c) The tube may be approximated by a uniform beam.

Earlier investigations have developed linearized mathematical models capable of describing the behavior of the tube up to and including the threshold of instability. Section 1 of Part II briefly discusses the approach taken by Alan S. Greenwald and John Dugundji (1). To predict the large

amplitude periodic motion after the threshold of instability is exceeded, nonlinearities have to be taken into consideration. One such nonlinearity is due to tension induced by bending and consequent stretching of the line. Its existence is closely examined in Part II, Section 1.2. So far, this study is believed to be the only one conducted on the nonlinear tension effect.

Computer simulation is used to solve the set of ordinary differential equations obtained by applying Galerkin's method to the partial differential equations of motion. Data have been analyzed and the results are presented graphically for two sets of tube boundary conditions, cantilevered (clamped-free) and simply supported-simply supported. The clamped-free tube undergoes limit-cycle oscillations of constant peak amplitude and frequency at a fixed fluid velocity. Unlike what was expected, the maximum stress does not always develop at the root of the cantilevered tube (the clamped end). In fact, the point of maximum stress travels along the length of the tube during one period of the limit cycle oscillation. The stress and deflection of the tube increases as fluid velocity increases. For a simply supported-simply supported tube, a static deflection occurs above the threshold of instability whose amplitude increases as the flow velocity increases. At still higher velocity, the tube enters into a dynamically unstable region.

PART II: CANTILEVERED TUBE

Section 1

Derivation of Equations of Motion

1.1 General Approach*

To obtain a differential equation of motion describing the tube, an energy approach has been utilized to account for all the major forces acting on the nonconservative system. Hamilton's principle states

$$\delta \int_{t_1}^{t_2} (T-U)\, dt + \int_{t_1}^{t_2} \delta W\, dt = 0 \tag{1}$$

The total kinetic energy of the system is the sum of the kinetic energy due to the motion of the line, and the kinetic energy due to the flowing fluid. In other words,

$$T = T_{line} + T_{fluid} \tag{2}$$

where

$$T_{line} = \frac{1}{2} \int_{o}^{\ell} m \left(\frac{\partial y}{\partial t}\right)^2 dx, \tag{3}$$

$$T_{fluid} = \frac{1}{2} \int_{o}^{\ell} m_f |\vec{V}|^2\, dx \tag{4}$$

In this paper, the x and y components of the fluid velocity are approximated as**

$$\left.\begin{aligned} V_x &\sim V \\ V_y &\sim \frac{\partial y}{\partial t} + V \frac{\partial y}{\partial x} \end{aligned}\right\} \tag{5}$$

---

* A similar discussion was previously reported in reference 1.

** See figure 3.

Then $$T_{fluid} = \frac{1}{2} \int_{o}^{\ell} m_f[V^2 + (\frac{\partial y}{\partial t} + V \frac{\partial y}{\partial x})^2]\ dx \tag{6}$$

and $$T = \frac{1}{2} \int_{o}^{\ell} \{m(\frac{\partial y}{\partial t})^2 + m_f[V^2 + (\frac{\partial y}{\partial t})^2 + 2V \frac{\partial y}{\partial t} \frac{\partial y}{\partial x} + V^2(\frac{\partial y}{\partial x})^2]\}\ dx \tag{7}$$

The total potential energy of the system consists of elastic strain energy of bending and the elastic energy stored in tension due to gravity. By assumption (c) of Part I, the concept of simple beam theory may be applied. In mathematical notation the total potential energy may be written as

$$U = \frac{1}{2} \int_{o}^{\ell} EI\ (\frac{\partial^2 y}{\partial x^2})^2\ dx + \frac{1}{2} \int_{o}^{\ell} mg(\ell-x)\ (\frac{\partial y}{\partial x})^2\ dx \tag{8}$$

A structural damping force and a shear force are assumed to be the two major nonconservative forces. Furthermore it is assumed that this structural damping is a viscous damping force of the form

$$\begin{aligned} f_{damp} &= D \frac{\partial y}{\partial t} \\ &= 2\zeta_1\ \bar{\omega}_1\ m \frac{\partial y}{\partial t} \end{aligned} \tag{9}$$

From momentum considerations, the shear force at the end of the line may be given as *

$$R = m_f\ V\ [\frac{\partial y}{\partial t}(\ell) + V \frac{\partial y}{\partial x}(\ell)] \tag{10}$$

It then follows that

$$\delta W = -\int_{o}^{\ell} 2\zeta_1\bar{\omega}_1\ m \frac{\partial y}{\partial t}\ \delta y\ dx - m_f\ [\frac{\partial y}{\partial t}(\ell) + V \frac{\partial y}{\partial x}(\ell)]\ \delta y(\ell) \tag{11}$$

---

* See Figure 3.

To derive the general equations of motion from Hamilton's Principle, equations (7), (8) and (1) are substituted into equation (1) first. The procedures of the calculus of variation are then performed. Appendix I details the mathematics involved. After considerable algebra, the partial differential equation of motion is found to be

$$EI \frac{\partial^4 y}{\partial x^4} + (m + m_f) \frac{\partial^2 y}{\partial t^2} + 2m_f V \frac{\partial^2 y}{\partial t \partial x} + m_f V^2 \frac{\partial^2 y}{\partial x^2}$$

$$+ 2\zeta_1 \bar{\omega}_1 m \frac{\partial y}{\partial t} - mg \frac{\partial}{\partial x} [(\ell - x) \frac{\partial y}{\partial x}] = 0 \tag{12}$$

and the boundary conditions are*

$$EI \frac{\partial^2 y}{\partial x^2} (\ell) = 0 \tag{13}$$

$$\frac{\partial y}{\partial x} (0) = 0 \tag{14}$$

$$EI \frac{\partial^3 y}{\partial x^3} (\ell) = 0 \tag{15}$$

$$y(0) = 0 \tag{16}$$

1.2 The Nonlinear Tension Effect**

A very important physical characteristic of the tube undergoing flutter is its large amplitude periodic motion. This can only be explained by taking nonlinear effects into account because no linearized model can predict such a periodic solution.

---

* In Part III these are changed to those appropriate for simply supported edges.

** Up until now, it has been believed that this is the only study on the nonlinear tension effect on the dynamics of a fluttering propellant line (tube).

Imagine that the tube is deflected as shown in the free body diagram for a differential length, ds (Fig. 4). For small displacements, the approximations

$$\theta_1 \sim \sin \theta_1 \sim \partial y/\partial x \tag{17}$$

$$\theta_2 \sim \sin \theta_2 \sim \frac{\partial}{\partial x} \left(\frac{\partial y}{\partial x}\right) dx \tag{18}$$

may be used. Summing forces in the y-direction due to tension yields a net force of

$$T_o \frac{\partial y}{\partial x} - T_o \left(\frac{\partial y}{\partial x} + \frac{\partial^2 y}{\partial x^2} dx\right) = -T_o \frac{\partial^2 y}{\partial x^2} dx \tag{19}$$

for a tube element of differential length, dx. Assuming that the propellant line is Hookean ideal elastic, then

$$T_o = \sigma_x a \tag{20}$$

$$E = \frac{\sigma_x}{\varepsilon_x} \tag{21}$$

where

$$\varepsilon_x = \frac{\Delta \ell}{\ell} \tag{22}$$

The elongated length can be written as

$$ds = \sqrt{(\partial x)^2 + (\partial y)^2} \tag{23}$$

and

$$\int ds = \int_o^\ell \sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2} dx$$

$$= \int_o^\ell \left[1 + \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2 + \ldots\right] dx \tag{24}$$

Neglecting higher order terms, equation (22) may be written as

$$\varepsilon_x = \frac{\int_o^\ell \left[1 + \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2\right] dx - \ell}{\ell} = \frac{1}{2\ell} \int_o^\ell \left(\frac{\partial y}{\partial x}\right)^2 dx \tag{25}$$

Combining equations (25), (21) and (20) give

$$T_o = \frac{Ea}{2\ell} \int_0^{\ell} \left(\frac{\partial y}{\partial x}\right)^2 dx \tag{26}$$

With this tension term and using (19) and (26), the equation of motion becomes

$$EI \frac{\partial^4 y}{\partial x^4} + (m+m_f) \frac{\partial^2 y}{\partial t^2} + 2m_f V \frac{\partial^2 y}{\partial t \partial x} + m_f V^2 \frac{\partial^2 y}{\partial x^2} +$$

$$2\zeta_1 \bar{\omega}_1 m \frac{\partial y}{\partial t} - mg \frac{\partial}{\partial x} \left[(\ell - x) \frac{\partial y}{\partial x}\right]$$

$$- \frac{Ea}{2\ell} \int_0^{\ell} \left[\left(\frac{\partial y}{\partial x}\right)^2 dx\right] \cdot \left[\frac{\partial^2 y}{\partial x^2}\right] = 0 \tag{27}$$

In most cases, the gravity force is small compared to the other forces and may be neglected. Thus,

$$EI \frac{\partial^4 y}{\partial x^4} + (m+m_f) \frac{\partial^2 y}{\partial t^2} + 2m_f V \frac{\partial^2 y}{\partial t \partial x} + m_f V^2 \frac{\partial^2 y}{\partial x^2} +$$

$$2\zeta_1 \bar{\omega}_1 m \frac{\partial y}{\partial t} - \frac{Ea}{2\ell} \left[\int_0^{\ell} \left(\frac{\partial y}{\partial x}\right)^2 dx\right] \frac{\partial^2 y}{\partial x^2} = 0 \tag{28}$$

Section 2

Methods of Solution

2.1 Non-dimensional Equations of Motion

The coordinate system (ξ, ϕ) and time τ chosen to nondimensionalize the equations of motion are defined as

$$\left.\begin{aligned} \xi &= x/\ell \\ \phi &= y/r, \ (r^2 = \bar{R}^2/2)^* \end{aligned}\right\} \qquad (29)$$

$$\tau = \sqrt{\frac{EI}{(m+m_f)\ell^4}}\ t \qquad (30)$$

With the above dimensionless quantities substituted into equation (28), the equation of motion is found to be

$$\frac{\partial^4\phi}{\partial\xi^4} + \frac{\partial^2\phi}{\partial\tau^2} + \frac{2Vm_f\ell}{\sqrt{EI(m+m_f)}}\frac{\partial^2\phi}{\partial\xi\partial\tau} + \frac{m_fV^2\ell^2}{EI}\frac{\partial^2\phi}{\partial\xi^2}$$

$$+ \frac{2\zeta_1\bar{\omega}_1 m\ell^2}{\sqrt{EI(m+m_f)}}\frac{\partial\phi}{\partial\tau} - \frac{ar^2}{2I}\int_0^1 \left(\frac{\partial\phi}{\partial\xi}\right)^2 d\xi\ \frac{\partial^2\phi}{\partial\xi^2} = 0 \qquad (30)$$

With $\beta \equiv \sqrt{\frac{m_f\ell^2}{EI}}\ V$ (velocity parameter)

$\mu \equiv \frac{m_f}{m + m_f}$ (mass ratio)

---

* See Appendix III for the derivation.

$$\nu \equiv \frac{2\zeta_1 \bar{\omega}_1 m\ell^2}{\sqrt{EI(m+m_f)}}\,^* \qquad \text{(damping parameter)}$$

and the fact that for a thin cylindrical tube, $I = ar^2$, equation (30) can be expressed as

$$\frac{\partial^4\phi}{\partial\xi^4} + \frac{\partial^2\phi}{\partial\tau^2} + 2\beta\sqrt{\mu}\,\frac{\partial^2\phi}{\partial\xi\partial\tau} + \beta^2\,\frac{\partial\phi}{\partial\tau} - \frac{1}{2}\int_0^1 \left(\frac{\partial\phi}{\partial\xi}\right)^2 d\xi \cdot \left[\frac{\partial^2\phi}{\partial\xi^2}\right] = 0 \qquad (31)$$

Equation (31) has associated boundary conditions

$$\text{at } \xi = 0 \qquad \phi = 0, \qquad \frac{\partial\phi}{\partial\xi} = 0 \qquad (32)$$

$$\text{at } \xi = 1 \qquad \frac{\partial^2\phi}{\partial\xi^2} = 0, \qquad \frac{\partial^3\phi}{\partial\xi^3} = 0 \qquad (33)$$

2.2 Galerkin's Method

The partial differential equation discussed in the previous section may be reduced to a set of ordinary differential equations by using Galerkin's method. It is assumed that the solution is of the form

$$\phi\,(\xi,\,\tau) = \sum_{k=1}^{m} \gamma_k\,(\xi)\,q_k\,(\tau) \qquad (34)$$

where $q_k\,(\tau)$'s are unknown functions of time, and $\gamma_k\,(\xi)$'s are assumed functions satisfying all boundary conditions of the problem. For the uniform cantilever beam considered in this study, $\gamma_k\,(\xi)$'s are taken to be

$$\gamma_k(\xi) = \cosh\lambda_k\xi - \cos\lambda_k\xi - \sigma_k[\sinh\lambda_k\xi - \sin\lambda_k\xi] \qquad (35)$$

Values of λ_k and σ_k are given in Table 1. Substituting equation (34) into equation (31), and using the notations

---

* $\bar{\omega}_1 = 3.52\sqrt{EI/m\ell^4}$

$$\frac{\partial q_k(\tau)}{\partial \tau} = \dot{q}_k(\tau) \qquad \frac{\partial \gamma_k(\xi)}{\partial \xi} = \gamma_k'(\xi) \tag{36}$$

The following equation is obtained

$$\sum_{k=1}^{m} q_k(\tau)\, \gamma_k^{iv}(\xi) + \sum_{k=1}^{m} \ddot{q}_k(\tau)\, \gamma_k(\xi) + 2\beta\sqrt{\mu} \sum_{k=1}^{m} \gamma_k'(\xi)\, \dot{q}_k(\tau) + \beta^2 \sum_{k=1}^{m} q_k(\tau)\, \gamma_k''(\xi)$$

$$+ \nu \sum_{k=1}^{m} \gamma_k(\xi)\, \dot{q}_k(\tau) - \frac{1}{2} \left[\int_0^1 \left(\sum_{k=1}^{m} q_k\, \gamma_k'(\xi)\right)^2 d\xi\right] \cdot \left[\sum_{k=1}^{m} q_k(\tau)\, \gamma_k''(\xi)\right] = 0 \tag{37}$$

The integral in the last term of equation (37) may be re-written as

$$\int_0^1 \left(\sum_{k=1}^{m} q_k(\tau)\, \gamma_k'(\xi)\right)^2 d\xi = \sum_{h=1}^{m} \sum_{i=1}^{m} \left[q_h(\tau)\, q_i(\tau) \cdot \int_0^1 \gamma_h'(\xi)\, \gamma_i'(\xi)\, d\xi\right] \tag{38}$$

The mode shape integral is evaluated in Appendix II with its values tabulated as c_{hi} in Table 2. Equation (38) then becomes

$$\int_0^1 \left(\sum_{k=1}^{m} q_k(\tau)\, \gamma_k'(\xi)\right)^2 d\xi = \sum_{h=1}^{m} \sum_{i=1}^{m} c_{hi}\, q_h(\tau)\, q_i(\tau) \tag{39}$$

Applying Galerkin's method to equation (37) yields

$$\sum_{k=1}^{m} \Big\{\Big[\int_0^1 \gamma_j \gamma_k^{iv}\, d\xi\Big] q_k + \Big[\int_0^1 \gamma_j \gamma_k\, d\xi\Big] \ddot{q}_k$$

$$+ 2\beta\sqrt{\mu} \Big[\int_0^1 \gamma_j \gamma_k'\, d\xi\Big] \dot{q}_k + \beta^2 \Big[\int_0^1 \gamma_j \gamma_k''\, d\xi\Big] q_k$$

$$+ \nu \Big[\int_0^1 \gamma_j \gamma_k\, d\xi\Big] \dot{q}_k - \frac{1}{2} \sum_{h=1}^{m} \sum_{i=1}^{m} q_h(\tau)\, q_i(\tau)\, c_{hi} \Big[\int_0^1 \gamma_j \gamma_k''\, d\xi\Big] q_k \Big\} = 0$$

$$(j = 1,2,\ldots, m) \tag{40}$$

To simplify equation (40), define

$$a_{jk} = \int_0^1 \gamma_j \gamma_k' \, d\xi \qquad (41)$$

$$b_{jk} = \int_0^1 \gamma_j \gamma_k'' \, d\xi \qquad (42)$$

a_{jk} and b_{jk} have been evaluated in the same manner as c_{hi}. Their values are also tabulated in Table 2. Furthermore,

$$\int_0^1 \gamma_j \gamma_k \, d\xi = \delta_{jk} \qquad (43)$$

$$\int_0^1 \gamma_j \gamma_k^{iv} \, d\xi = \delta_{jk} \lambda_k^4 \qquad (44)$$

where δ_{jk} is the delta function. Equation (40) may be further reduced to

$$\sum_{k=1}^{m} \{\delta_{jk} \ddot{q}_k + [2\beta \sqrt{\mu}\, a_{jk} + \nu\delta_{jk}] \dot{q}_k$$

$$+ [\delta_{jk} \lambda_k^4 + \beta^2 b_{jk} - \frac{1}{2} \sum_{h=1}^{m} \sum_{i=1}^{m} q_h q_i c_{hi} b_{jk}] q_k\} = 0$$

$$(j = 1,2,\ldots,m) \qquad (45)$$

2.3 Numerical Methods

The time history of the tube motion may be determined by solving equation (45) numerically. For simplicity, first consider a two mode analysis for equation (45) without the nonlinear term, that is

$$\sum_{k=1}^{2} \{\delta_{jk}\, \ddot{q}_k + [2\beta \sqrt{\mu}\, a_{jk} + \delta_{jk}\, \nu]\, \dot{q}_k + [\delta_{jk}\lambda_k{}^4 + \beta^2\, b_{jk}]\, q_k\} = 0$$

$$(j = 1,2) \tag{46}$$

In another form,

$$\ddot{q}_1 + [2\beta \sqrt{\mu}\, a_{11} + \nu]\, \dot{q}_1 + [\lambda_1{}^4 + \beta^2\, b_{11}]\, q_1 + 2\beta \sqrt{\mu}\, a_{12}\, \dot{q}_2 + \beta^2\, b_{12}\, q_2 = 0 \tag{47}$$

$$2\beta \sqrt{\mu}\, a_{21}\, \dot{q}_1 + \beta^2\, b_{21}\, q_1 + \ddot{q}_2 + [2\beta \sqrt{\mu}\, a_{22} + \nu]\, \dot{q}_2 + [\lambda_2{}^4 + \beta^2\, b_{22}]\, q_2 = 0 \tag{48}$$

If one defines

$$q_3 = \dot{q}_4 \quad \Rightarrow \quad q_1 = \dot{q}_3 \tag{49}$$

$$q_4 = \dot{q}_2 \quad \Rightarrow \quad \ddot{q}_2 = \dot{q}_4 \tag{50}$$

equations (47) and (48) may be written in matrix form as

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -(\lambda_1^4+\beta^2 b_{11}) & -\beta^2 b_{12} & -(2\beta\sqrt{\mu}\, a_{11}+\nu) & -2\beta\sqrt{\mu}\, a_{12} \\ -\beta^2 b_{21} & -(\lambda_2^4+\beta^2 b_{22}) & -2\beta\sqrt{\mu}\, a_{21} & -(2\beta\sqrt{\mu}\, a_{22}+\nu) \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \tag{51}$$

In short, $$\dot{\vec{q}} = A\, \vec{q} \tag{52}$$

But $$\dot{\vec{q}}(\tau) \simeq \frac{\vec{q}(\tau+\Delta\tau) - \vec{q}(\tau)}{\Delta\tau} \tag{53}$$

then $$\vec{q}(\tau+\Delta\tau) = A\vec{q}(\tau)\, \Delta\tau + \vec{q}(\tau) \tag{54}$$

for $\Delta\tau$ sufficiently small.

The fact that equation (54) is easily expandable for an m-mode analysis makes it the preferred form for writing the equations of motion.

In the case of four mode nonlinear analysis, the nonlinear term of equation (45) alone generates 256 terms. However, only 20 of them are unique. The size of the matrix can thus be reduced to (8,28).*

The error of the approximated solution can be reduced by using the more accurate finite difference relation.**

$$q(\tau+\Delta\tau) \simeq q(\tau) + \Delta\tau(1 + \frac{1}{2}\nabla + 5/12\ \nabla^2 + \ldots)\ \dot{q}(\tau) \qquad (55)$$

where
$$\nabla\dot{q}(\tau) = \dot{q}(\tau) - \dot{q}(\tau-\Delta\tau) \qquad (56)$$

and
$$\nabla^{r+1}\ \dot{q}(\tau) = \nabla^r\ \dot{q}(\tau) - \nabla^r\ \dot{q}(\tau-\Delta\tau) \qquad (57)$$

A listing of WATFIV computer programs for four mode linear and nonlinear analysis is given in Appendix IV. They are based on the algorithms discussed in this sections and improved by equation (55). Initial conditions and data used in the program are given in Table 3.

The above approach to determination of nonlinear limit cycle oscillations is similar to that employed by Dowell for plates and shells (7).

---

* Note that the vector and the matrix have to be conformal.

** The formulas given can be found in Reference 6.

Section 3

Numerical Results

3.1 Comparison of Numerical and Previous Solutions

The three most interesting questions posed are, for a given mass ratio and damping ratio,

i) What is the fluid velocity at which the tube begins to flutter?

ii) What is the flutter frequency? and,

iii) What is the limit cycle amplitude of vibration?

Answers to all the questions above may be obtained from the raw computer data. The most primitive method is the "binary search" method.* The computer program is run at several values of the fluid velocities until the flutter behavior is observed. Binary search is then applied to determine the fluid velocity at which the system is neutrally stable.

Another method is to process the raw data. Frequencies and damping ratios of the different vibration modes are plotted against the nondimensional fluid velocity on the same graph. Flutter velocity and frequency may be readily detected from the graph. Examples of two and four-mode linear analysis are shown in Figures 5 and 6.

To make sure that the numerical results obtained reflect the actual dynamic instabilities of the tube, rather than numerical instabilities, the results from the four mode linear analysis are double checked against the previous solutions derived by Alan S. Greenwald and John Dugundji (1). Although the approaches used are significantly different, the two results are found to be in very close agreement. They are shown in Figures 7a and 7b.

---

* Concept of Binary Search is given in detail in Reference 8.

3.2 Theoretical Limit Cycle Predictions

From the nonlinear model, one can predict the behavior of a cantilevered tube conveying fluid at low fluid velocity. For a given mass ratio, the amplitude of the vibrations is expected to decay to zero with time. However, above a certain critical velocity, the amplitude would increase first as if there is no upper bound. After a few cycles, the amplitude of the vibrations drops back by a fraction and stays at a constant magnitude indefinitely. This is called a limit cycle. The amplitude of the limit cycle increases with the fluid velocity. Graphical presentations of the relationship between the amplitude and the fluid velocity are given in Figure 8 and 9 for mass ratios of 0.1 and 0.5 respectively.

First and second mode frequencies are identical at the threshold of instability. Beyond that, both frequencies increase with the fluid velocity. This is shown in figures 10 and 11 for mass ratios of 0.1 and 0.5 respectively.

3.3 Mode Shapes and Stress Distributions

Mode shapes of the fluttering tube can be obtained by holding τ in equation (34) constant. Altogether five values of τ are judiciously chosen to represent equal time increments over one-half of the period of the limit cycle oscillations. The mode shapes of the tube at these instances are plotted on the same scale for ease of comparison. Figures 12, 13, 14 show mode shapes of tube of mass ratio 0.1 for three different fluid velocity, β. Figures 15, 16, 17 show the cases where mass ratio is 0.5, and the mode shapes are at 1/2, 5/8, 3/4, 7/8 and the end of the period.

Stress distribution may be determined by using the following equation

$$\sigma_x = Er \frac{\partial^2 y}{\partial x^2} * \tag{58}$$

or $$\sigma_x = \frac{Er^2}{\ell^2} \frac{\partial^2 \phi}{\partial \xi^2}$$

$$= \frac{Er^2}{\ell^2} \sum_{k=1}^{m} q_k(\tau)\, \gamma_k''(\xi)$$

$$= \frac{Er^2}{\ell^2} \sum_{k=1}^{m} q_k(\tau) \cdot \lambda_k^2 \,[\cosh \lambda_k\xi + \cos \lambda_k\xi - \sigma_k \,(\sinh \lambda_k\xi + \sin \lambda_k\xi)] \tag{59}$$

Figure 18 shows the stress distribution in the tube for $\mu = 0.5$ and $\beta = 10.4$ at five different time steps. Unlike what was expected, the maximum stress does not occur at the root of the tube (the clamped end). It appears to be shifting along the length of the tube during the entire cycle of oscillation. This is because the maximum curvature of the tube does not necessarily occur at the root of the tube.

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* σ_x is in psi in Fig. 18.

PART III: SIMPLY SUPPORTED TUBE

Section 1

Equations of Motion

The partial differential equation of motion of a simply supported tube is the same as the one for a cantilever tube, i.e., equation 37. The only difference is the set of boundary conditions

$$\left.\begin{aligned} \text{at} \quad \xi = 0, \quad & \phi = 0 \\ & \frac{\partial^2 \phi}{\partial \xi^2} = 0 \end{aligned}\right\} \tag{60}$$

$$\left.\begin{aligned} \text{at} \quad \xi = 1, \quad & \phi = 0 \\ & \frac{\partial^2 \phi}{\partial \xi^2} = 0 \end{aligned}\right\} \tag{61}$$

It is again assumed that the solution is of the form

$$\phi(\xi,\tau) = \sum_{n=1}^{m} \gamma_n(\xi)\, q_n(\tau) \tag{62}$$

To satisfy the boundary conditions, the $\gamma_n(\xi)$'s are taken to be

$$\gamma_n(\xi) = \sin n\pi\xi \tag{63}$$

$$n = 1, \ldots, m$$

Since everything else is the same as for the cantilevered tube except for $\gamma_n(\xi)$, equation (40) can be reused. The only things that have to be rederived are the mode shape integrals. Mode shape integrals for a tube simply-supported at both ends are evaluated in Appendix V.

As before, define

$$a_{jk} = \int_0^1 \gamma_j \gamma_k' \, d\xi \tag{64}$$

$$b_{jk} = \int_0^1 \gamma_j \gamma_k'' \, d\xi \tag{65}$$

$$c_{hi} = \int_0^1 \gamma_h' \gamma_i' \, d\xi \tag{66}$$

and the equations of motion become

$$\sum_{k=1}^{m} \{\frac{1}{2} \delta_{jk} \ddot{q}_k + [2\beta \sqrt{\mu} \, a_{jk} + \frac{1}{2} \nu \delta_{jk}] \dot{q}_k$$

$$+ [\frac{k^4\pi^4}{2} \delta_{jk} + \beta^2 b_{jk} - \frac{1}{2} \sum_{h=1}^{m} \sum_{i=1}^{m} q_h q_i c_{hi} b_{jk}] q_k\} = 0 \tag{67}$$

$$(j = 1,2, \ldots , m)$$

The computer program written for evaluating the time histories of a cantilevered tube may be readily modified to solve equation (67) by the following changes

$$\delta_{jk} = \begin{cases} \frac{1}{2} & j=k \\ 0 & j \neq k \end{cases} \tag{68}$$

$$\lambda_k = k\pi \tag{69}$$

A listing of the modified program in FORTRAN is given in Appendix VI. Other alternations were made to conserve core storage space and computer time.

Section 2

Predicted Characteristics

For $0 \leq \beta < \pi$, the tube undergoes damped stable vibrations. Examples of these damped vibrations are shown in Figures 19 and 20 for the two limiting cases. The magnitude of the true damping ratio, ζ_1^T, increases with flow velocity. This relation is shown in Figure 21a. First mode frequency, ω_1, as a function of β is shown in Figure 21b. The frequency of the first mode vibration decreases as β is increased π. In the neighborhood of $\beta = \pi$, the first mode damping ratio becomes very large and the frequency approaches zero. In Figure 22, the product $\zeta_1\omega_1$ is shown. Note that it varies smoothly with β as β passes through π.

Beyond $\beta = \pi$, the tube was found to undergo divergence by previous investigators, Ref. 5. This is only true for β less than 2π. For $\pi < \beta < 2\pi$, the magnitude of $q_1(\tau)$ grows exponentially with time initially. $q_1(\tau)$ then oscillates near some constant value, say q^*. These transient oscillations decay with time and eventually die out, and $q_1(\tau)$ converges to the steady state amplitude, i.e., q^*. Higher mode oscillations are not very significant and the amplitudes decay to zero. Examples of such behavior for four different values of β are given in Figures 23, 24, 25 and 26. To show the relationship between β and the amplitude of $q_1(\tau)$ more clearly, the non-oscillatory average of $q_1(\tau)$ is plotted against time for the four β's in Figure 27.

Near $\beta = 2\pi$, all four modes become dynamically unstable. However, the first and second modes are dominant. Four of the relevant time histories are illustrated in Figures 28 to 31.

It was speculated that if at $\beta = \pi$, the first mode becomes statically unstable, and at $\beta = 2\pi$, the second mode becomes important and changes the static instability to a dynamic instability, then at $\beta = 3\pi$, the third mode may become very significant and drastically alter the time history of motion again. However, this is not true. The third and fourth modes are dynamically unstable, but they do not dominate the total motion. For $\beta > 3\pi$, no drastic change in time history is found. It appears to have the same "pulse" or "beat" characteristics as for $\beta < 3\pi$. These are shown in Figures 32 and 33.

A stress analysis was done for $\beta = 5.92$. As was expected, the maximum stress is in the middle between the two ends. Figure 34 shows the stress distribution in the tube.

Although there is no drastic change in the time history for $\beta > 3\pi$, the third mode does appear in the deformation shape during the cycle. At β = 10.0, the deformation shape of the tube changes from a two mode sine wave to a three mode sine wave and back to a two mode sine wave over a complete cycle of motion. Deformation shapes at 20 equal time increments over the period of the modulated sine wave are shown in Figures 35a-35e. Graphs showing stress distribution in the tube for the 20 corresponding deformation shapes are given in Figures 36a-36e.

The effects of fluid velocity β on the first and second mode peak limit cycle amplitudes are shown in Figure 37a and 37b respectively. These further illustrate the dramatic changes in modal content of the motion which occur near $\beta = 2\pi$.

The first correct linear stability analysis of a simply supported-simply supported pipe was given by Housner (9). The present discussion extends his results to the nonlinear regime.

PART IV: CONCLUDING REMARKS

This study has developed a mathematical model capable of describing the large amplitude periodic behavior of a cantilever or simply-supported propellant line (tube) conveying high velocity liquid. The study has also illustrated the importance of the nonlinear tension effect induced by bending of the tube. Thresholds of instability predicted by the theory are in good agreement with those previously obtained by other investigators. Plans have been made for setting up experiments to confirm the predicted magnitudes of the limit cycles.

The theoretical model might be improved by considering other nonlinearities, e.g., nonlinear modulus of elasticity or nonlinear curvature. Instead of viscous damping, one might consider viscoelastic damping, or better still, a combination of viscous and viscous-elastic damping. The uniform beam assumption could be replaced by the thin cylindrical shell approximation. For cases in which the hanging tubes are very flexible, the gravity term should not be neglected.

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Table 1

MODE SHAPE PARAMETERS

(Cantilever Beam)

Mode	σ_k	λ_k	λ_k^4
1	0.73410	1.8751	12.362
2	1.01847	4.6941	485.52
3	0.99923	7.8548	3806.5
4	1.000034	10.9955	14617.

Table 2

MODE SHAPE INTEGRALS

(Cantilever Beam)

$$a_{jk} = \int_0^1 \gamma_j(\xi)\ \gamma_k'(\xi)\ d\xi$$

j/k	1	2	3	4
1	2.0000	-4.7600	3.7840	-4.1200
2	0.7595	2.0000	-6.2220	3.3830
3	0.2157	2.2220	2.0000	-8.1680
4	0.1198	0.6166	4.1680	2.0000

$$b_{jk} = \int_0^1 \gamma_j(\xi)\ \gamma_k''(\xi)\ d\xi$$

j/k	1	2	3	4
1	0.8582	-11.7400	27.4500	-37.3900
2	1.8740	-13.3000	-9.0420	30.4000
3	1.5650	3.2290	-45.9000	-8.3350
4	1.0870	5.5410	4.2540	-98.9200

$$c_{hi} = \int_0^1 \gamma_h'(\xi)\ \gamma_i'(\xi)\ d\xi$$

h/i	1	2	3	4
1	4.6478	-7.3799	3.9415	-6.5934
2	-7.3799	32.4173	-22.3524	13.5826
3	3.9415	-22.3542	77.2989	-35.6482
4	-6.5934	13.5826	-35.6482	142.4605

Table 3

INITIAL CONDITIONS AND DATA USED

$\vec{q} = 0.1$

$\vec{q} = 0$

$\ell = 9.5$ ins.

$m = 1.21 \times 10^{-6}$ lb-sec^2/in^2

$I = 46.6 \times 10^{-6}$ in^4

$\overline{\omega}_1 = 25.1$ rad/sec

$E = 1.07 \times 10^4$ lbs/in^2

$\zeta_1 = .080$

$\nu = .400$

$\mu = 0.1, 0.5$

Table 4

MODE SHAPE INTEGRALS

(Simply Supported Beam)

$$a_{jk} = \int_0^1 \gamma_j(\xi)\, \gamma_k'(\xi)\, d\xi$$

j/k	1	2	3	4
1	0.0000	-1.3333	0.0000	-0.5333
2	1.3333	0.0000	-2.4000	0.0000
3	0.0000	2.4000	0.0000	-3.4286
4	0.5333	0.0000	3.4286	0.0000

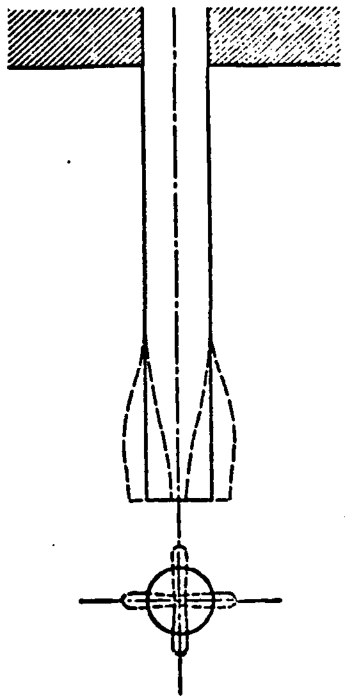


Figure 1. Observed Flutter of Short Tube
(From Reference 3)

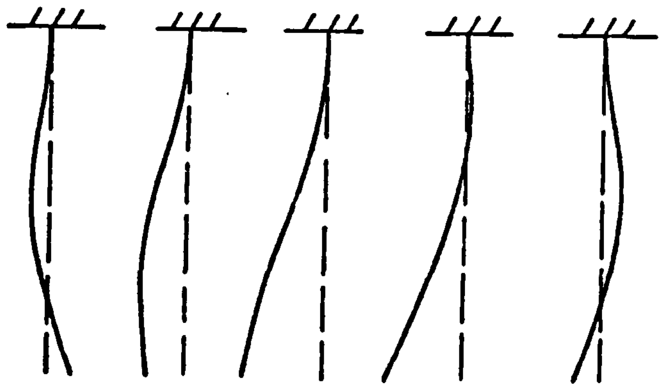
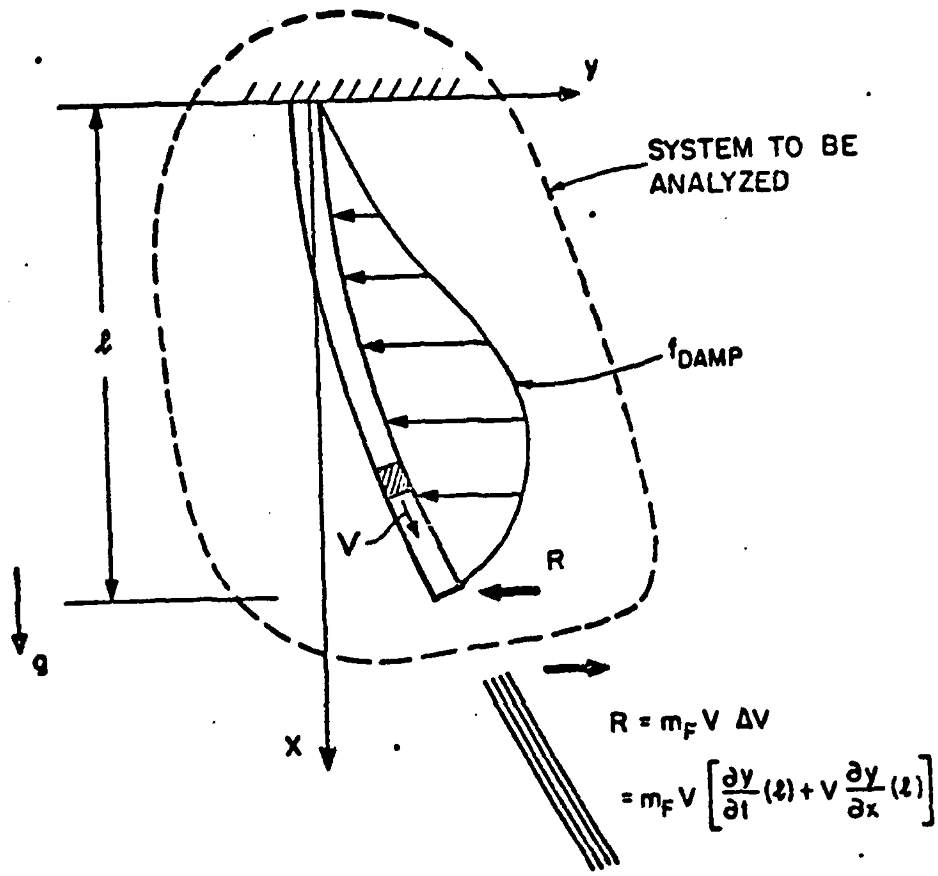


Figure 2. Flexural Oscillatory Instabilities
(From Reference 1)



ABSOLUTE VELOCITY OF FLUID,

$V_x \approx V$

$V_y \approx \frac{\partial y}{\partial t} + V \frac{\partial y}{\partial x}$

Figure 3. Tube with Flowing Fluid
(From Reference 1)

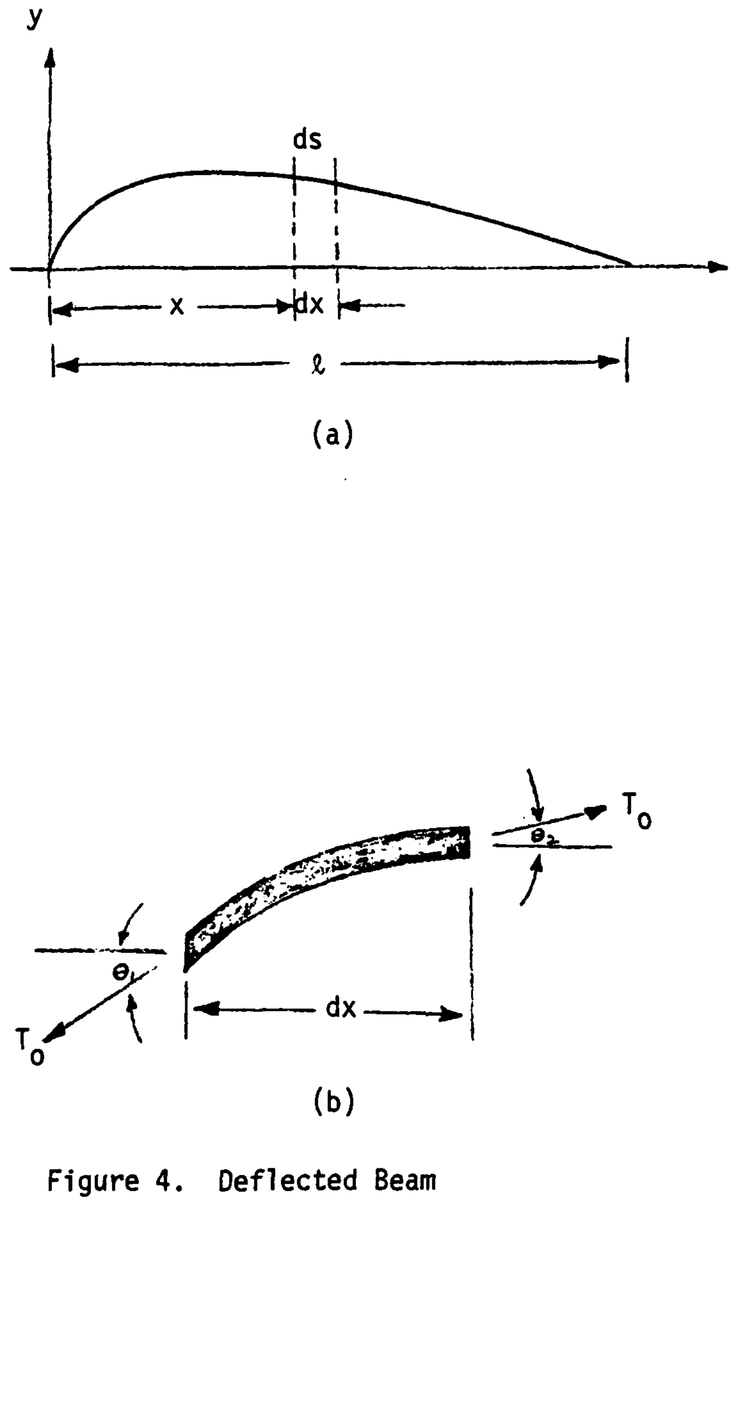


Figure 4. Deflected Beam

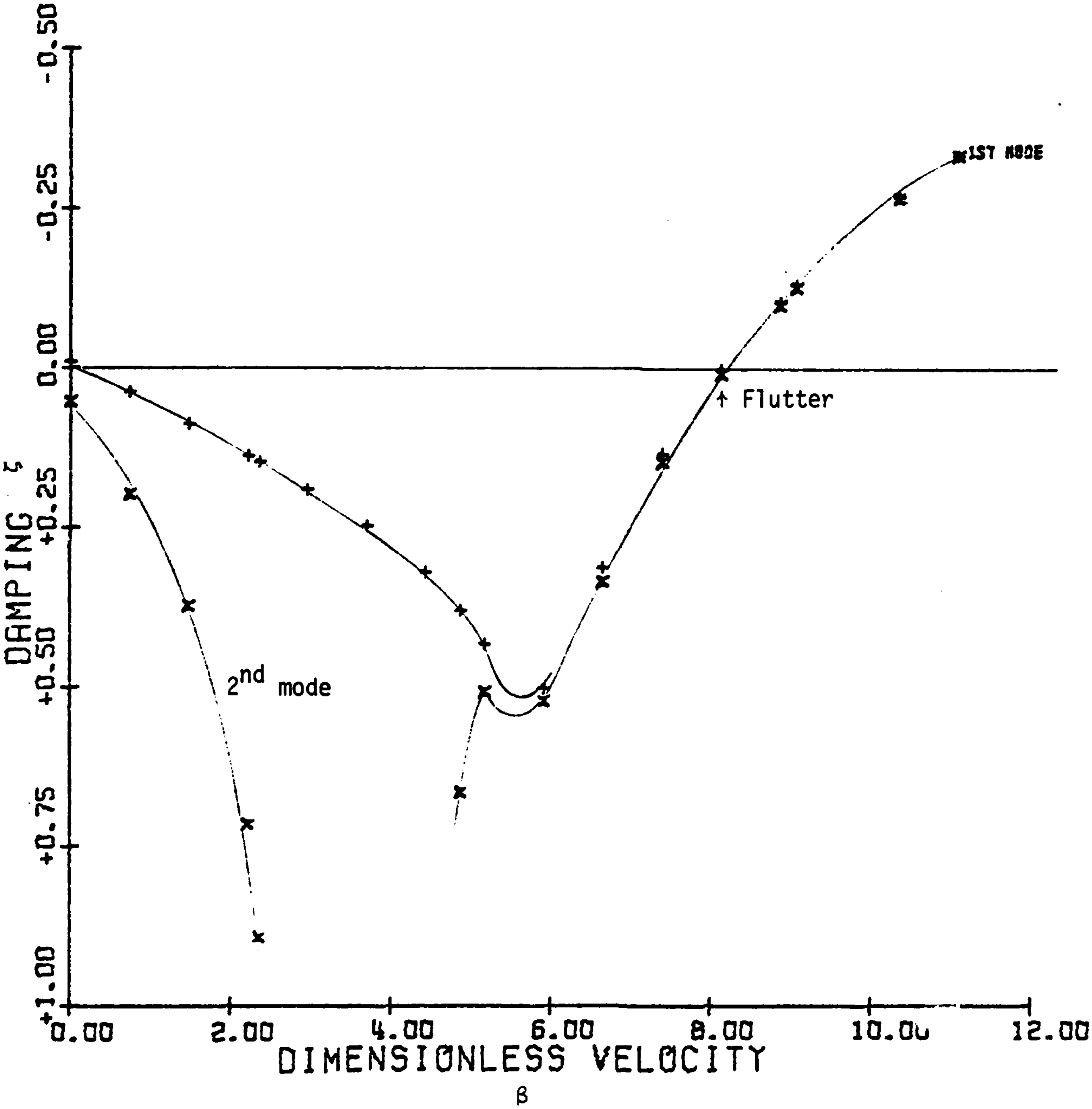


Figure 5. The Effects of Velocity on Damping (μ = 0.5)
(Two Modes)

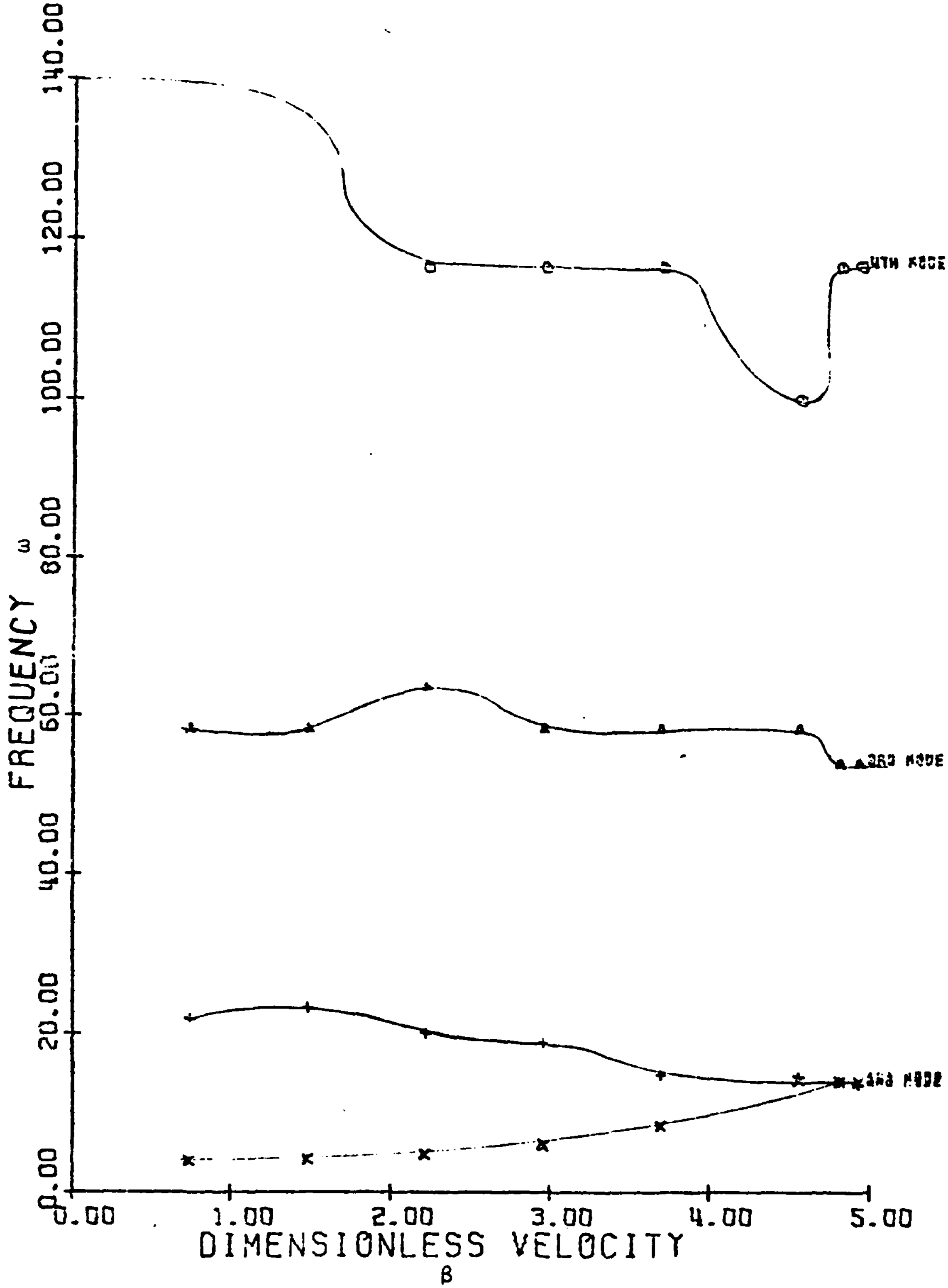


Figure 6a. The Effects of Velocity on Frequency (μ = 0.1)
(Two Modes)

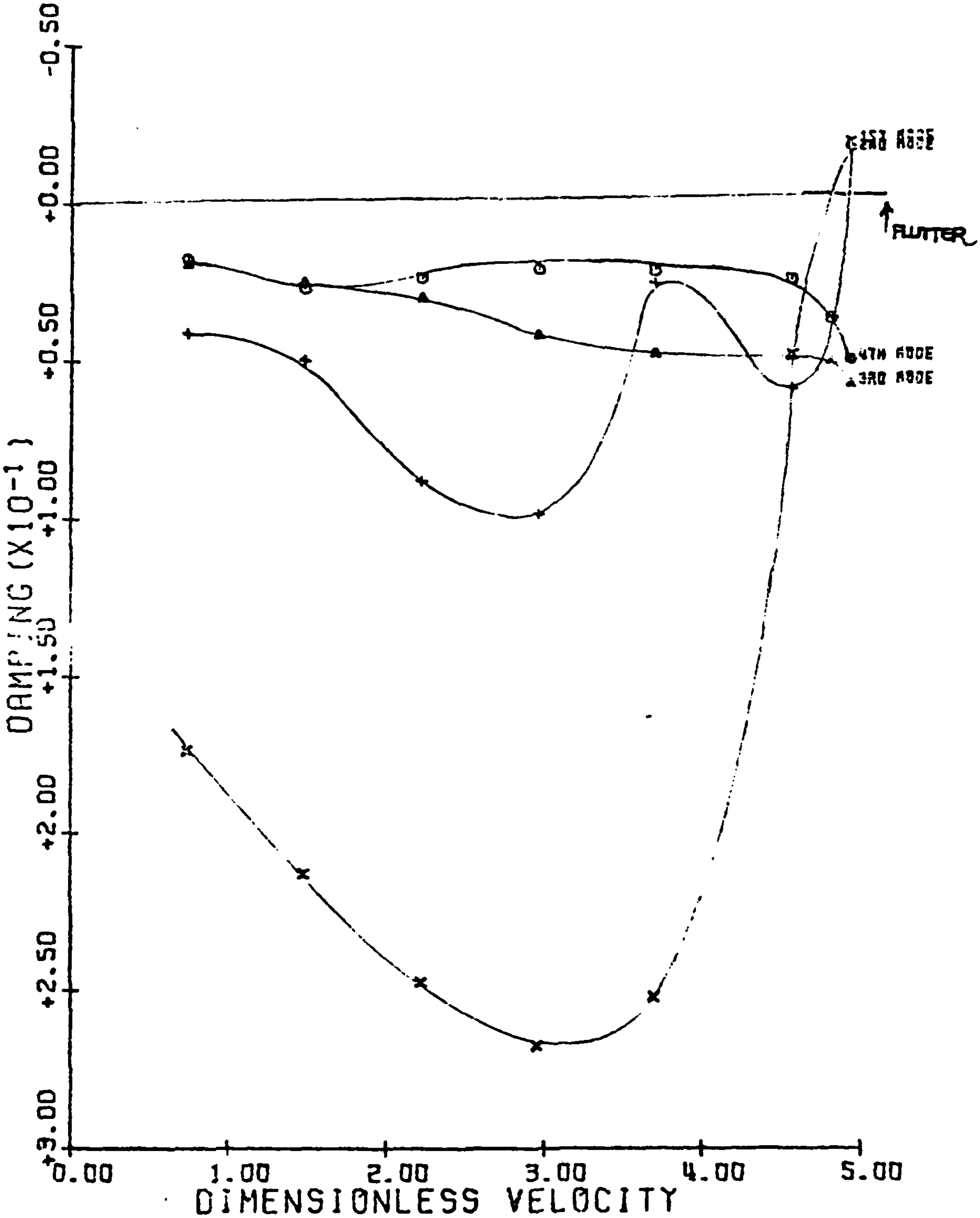


Figure 6b. The Effects of Velocity on Damping
(μ = 0.1, Four Modes)

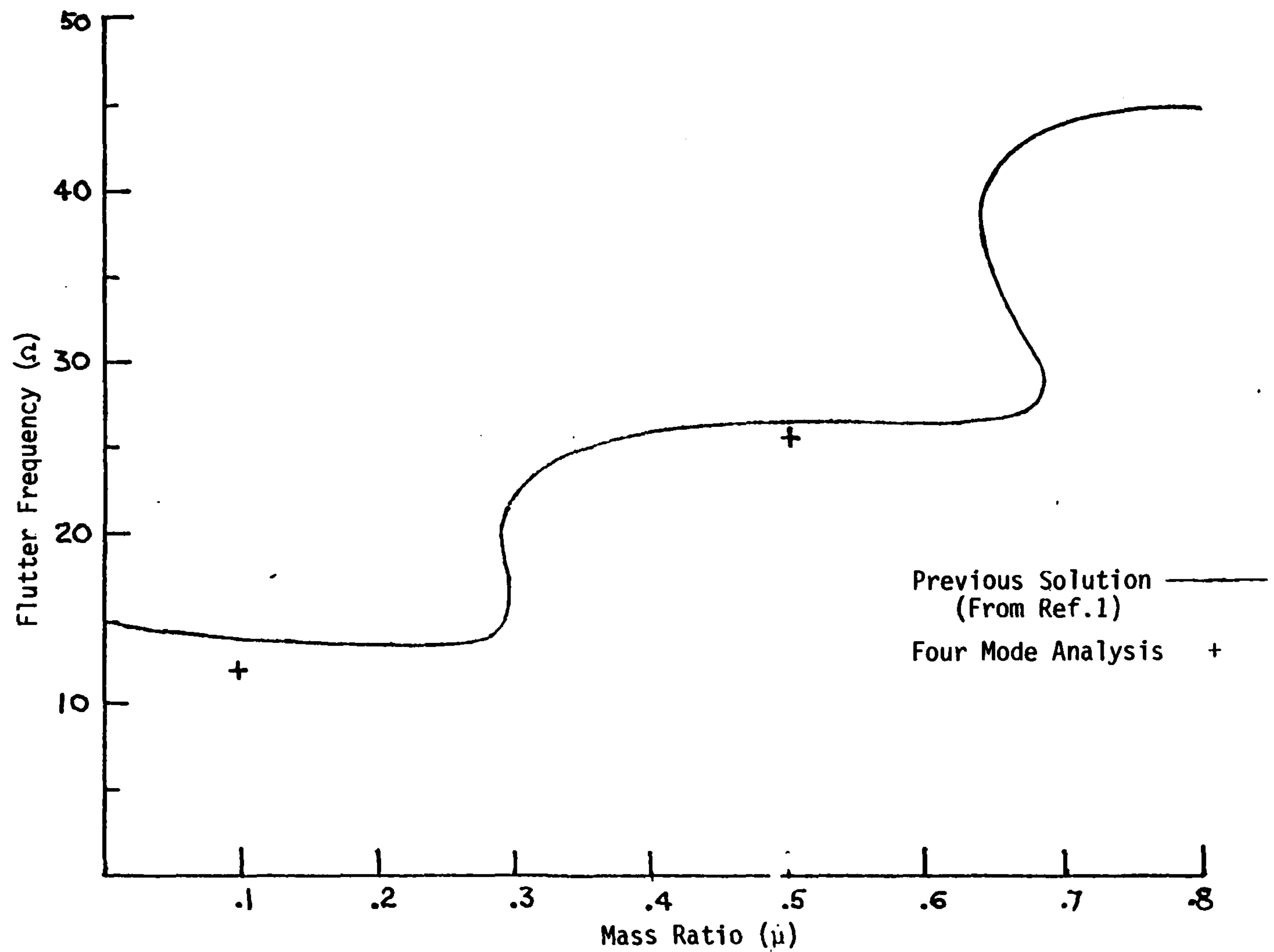


Figure 7a. Comparison of Previous and Numerical Solutions
(Frequency vs. Mass Ratio)

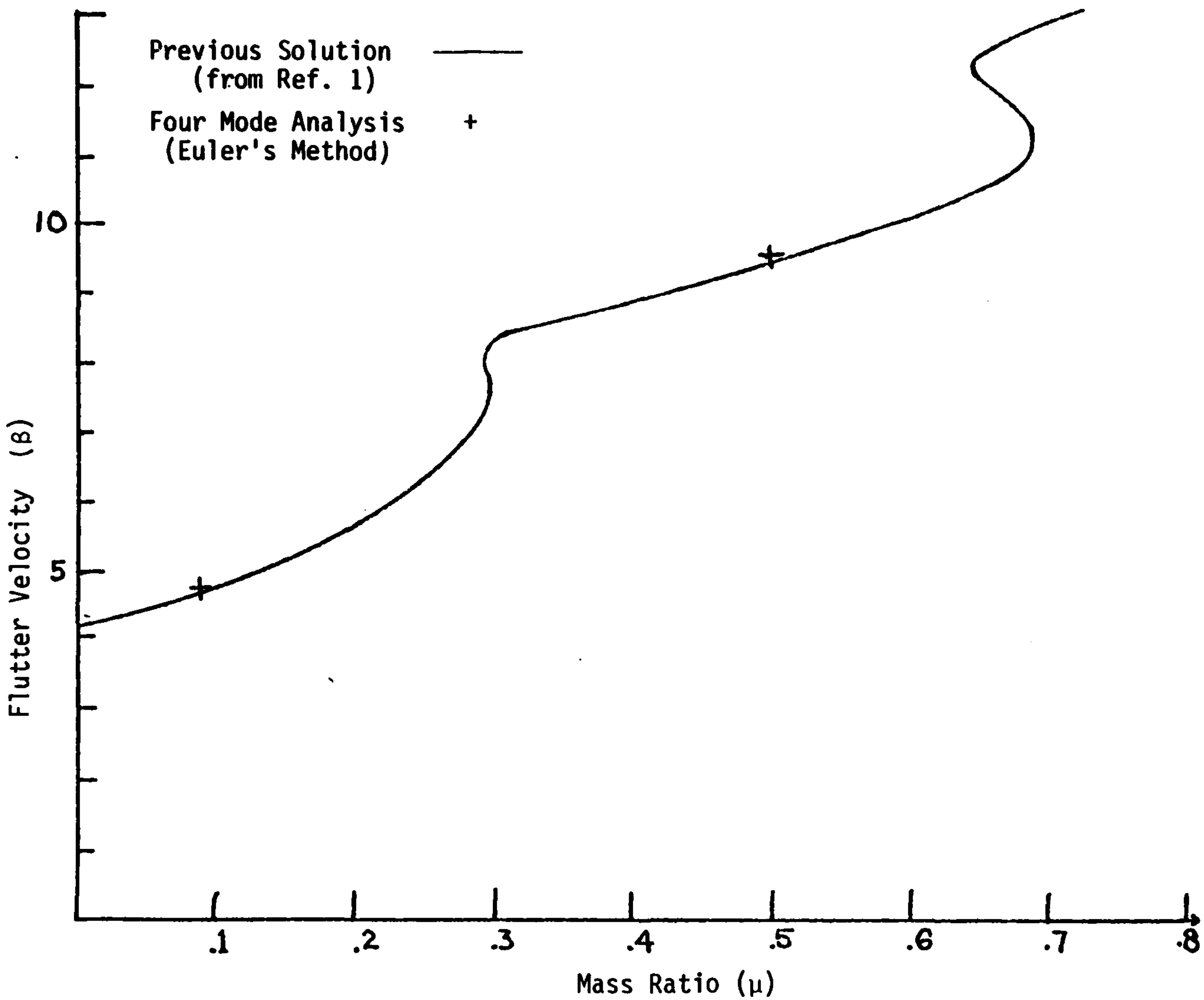


Figure 7b. Comparison of Previous Solution and Numerical Solutions (Velocity vs. Mass Ratio)

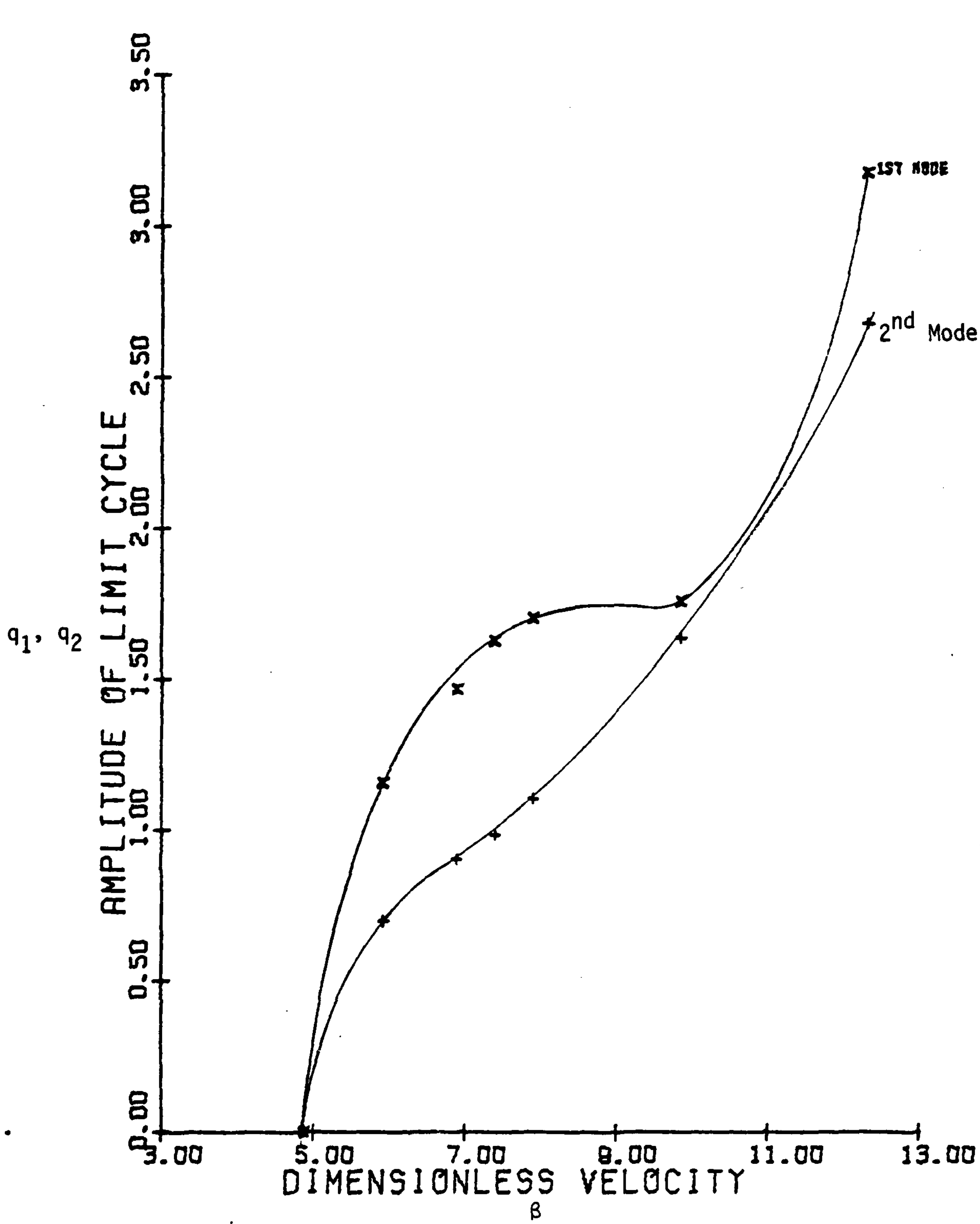


Figure 8. Effects of Velocity on Amplitude of Limit Cycle ($\mu = 0.10$, Four Modes)

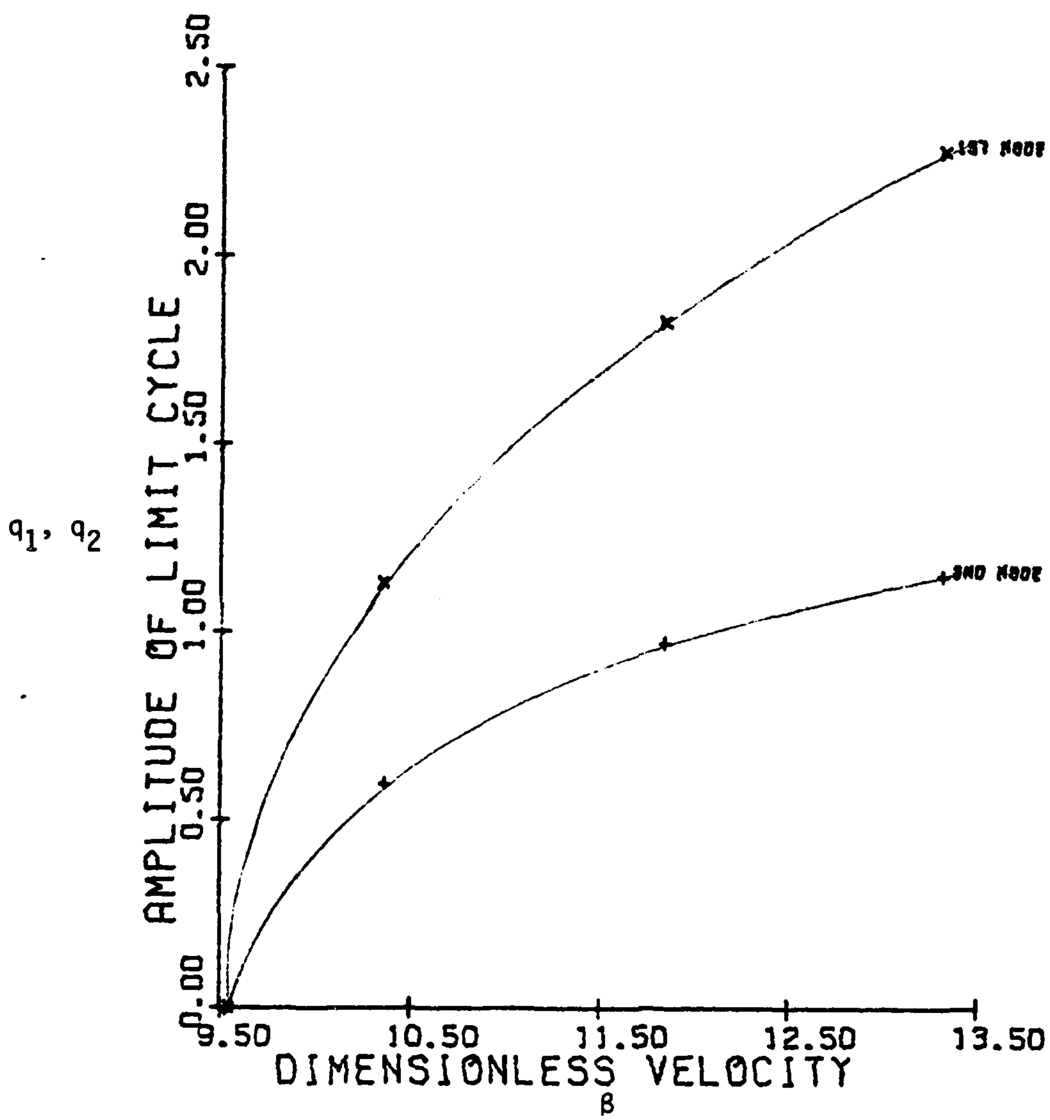


Figure 9. The Effects of Velocity on the Amplitude of Limit Cycle (μ = 0.50, Four Modes)

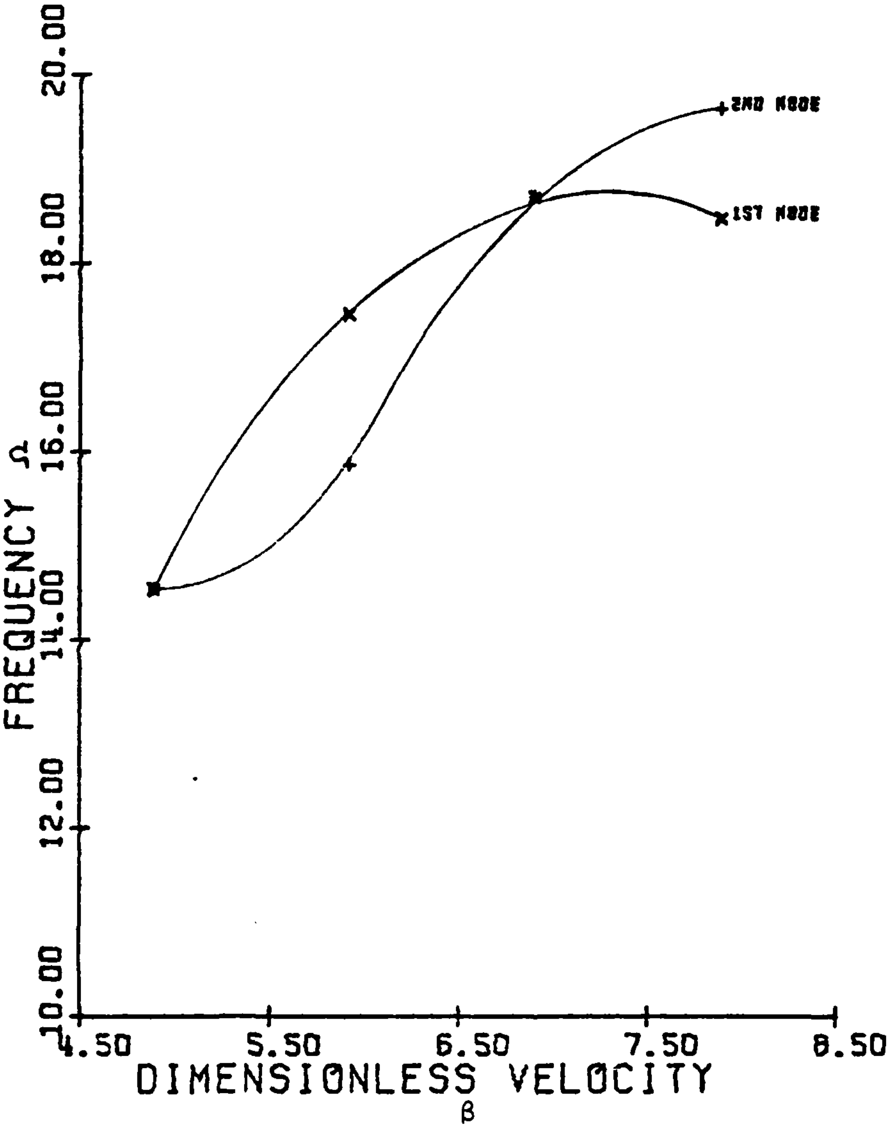


Figure 10. Variation of Frequency Due to Velocity

(μ = 0.1, Four Modes)

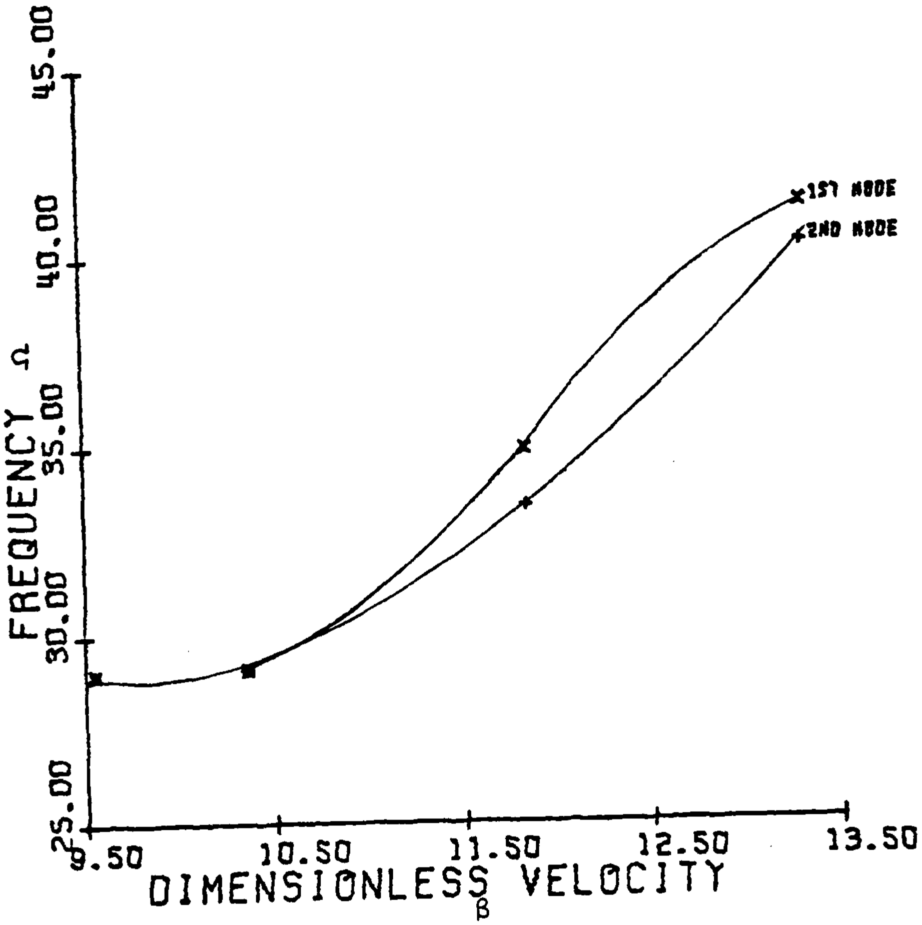


Figure 11. Variation of Frequency Due to Velocity
(μ = 0.5, Four Modes)

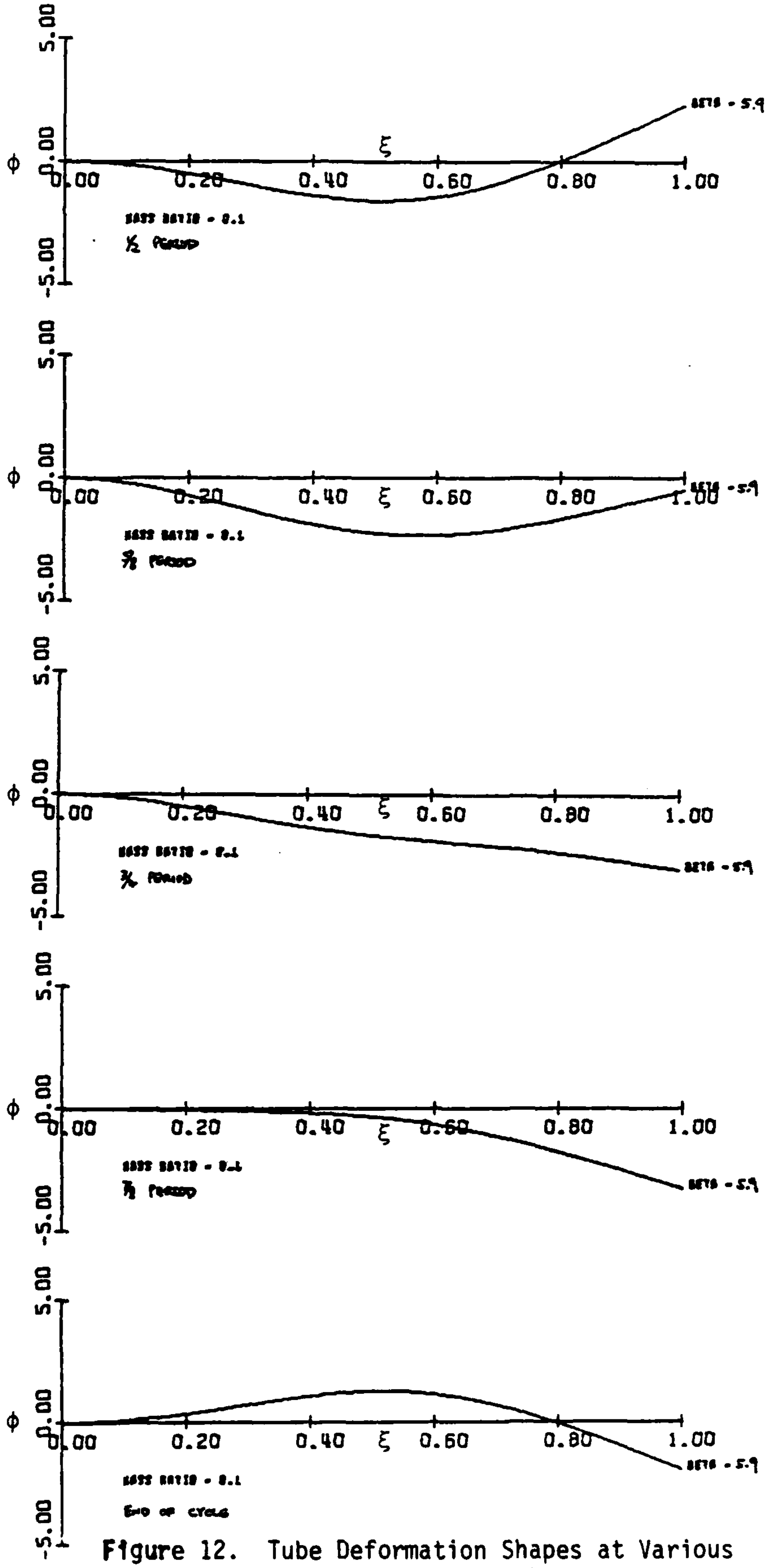


Figure 12. Tube Deformation Shapes at Various Times During Limit Cycle

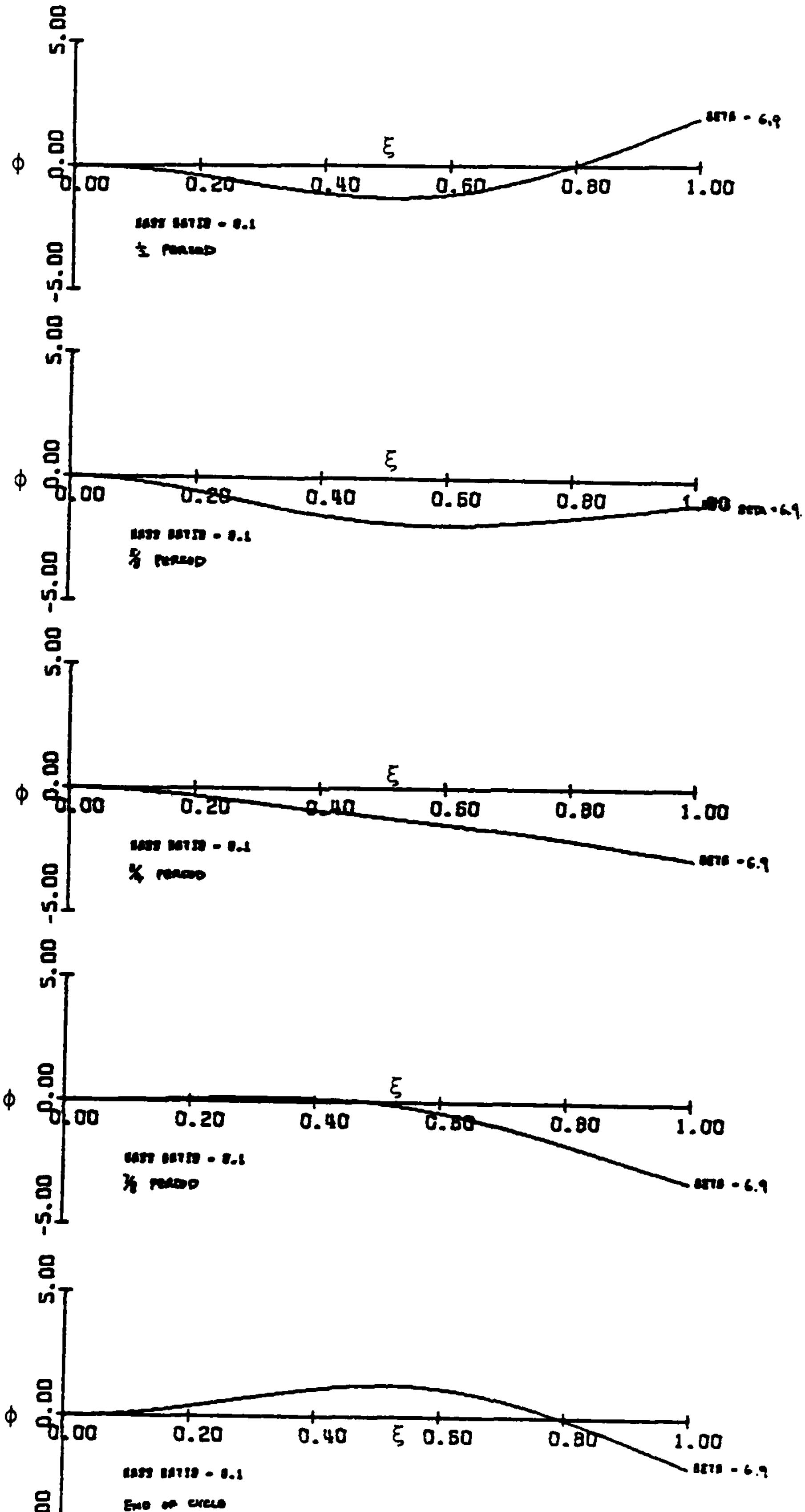


Figure 13. Tube Deformation Shapes at Various Times During Limit Cycle

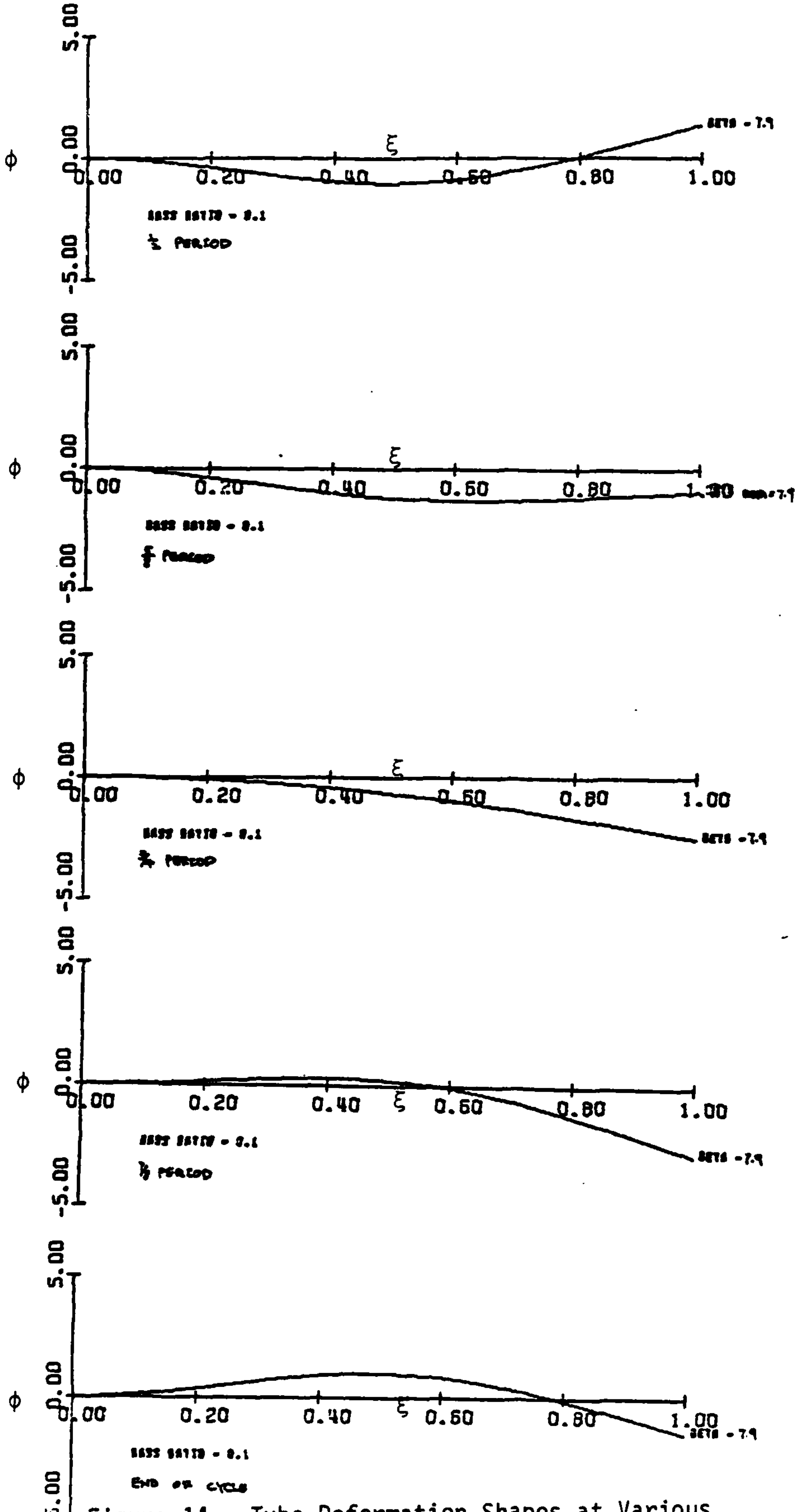


Figure 14. Tube Deformation Shapes at Various Times During Limit Cycle

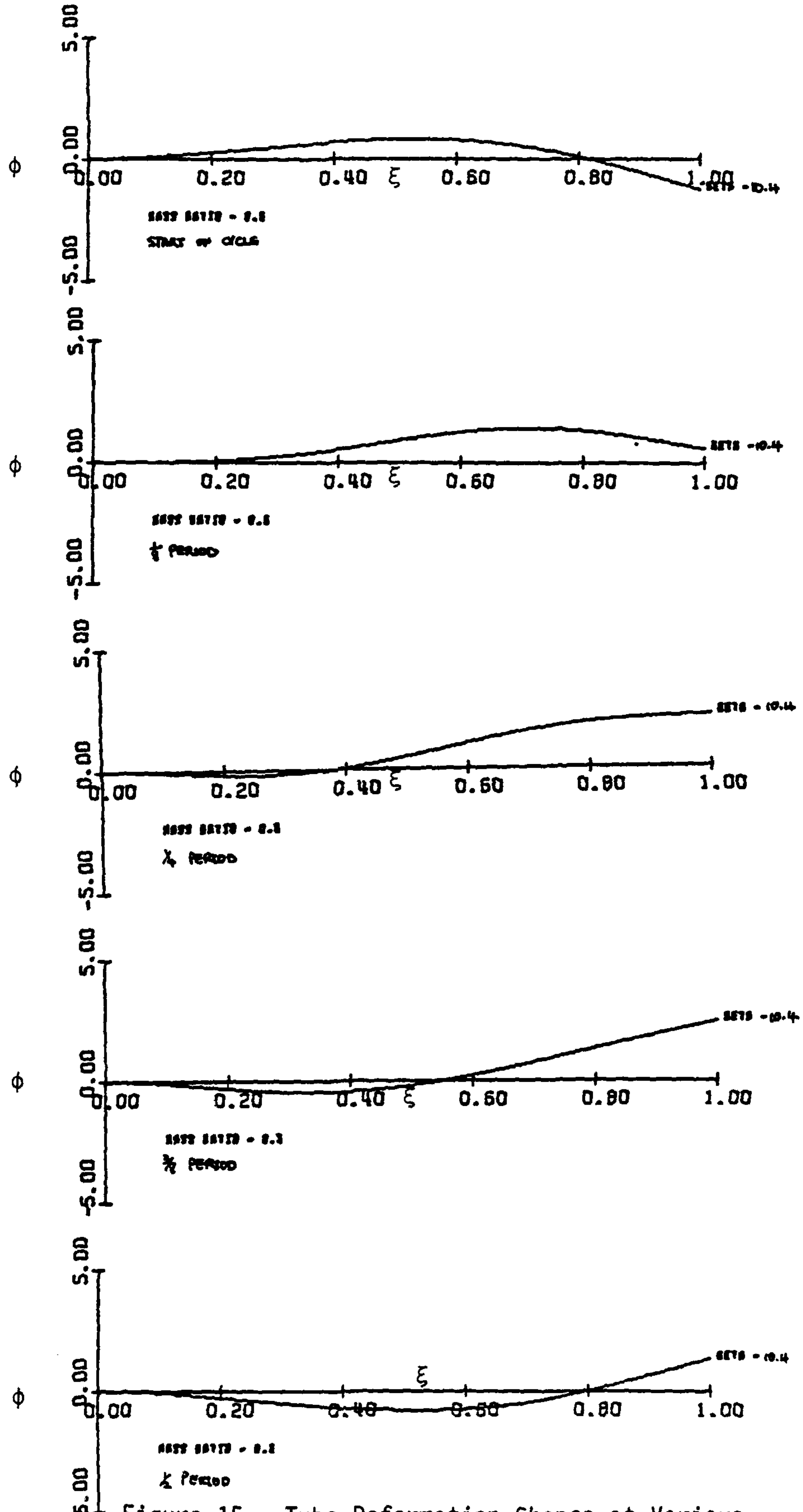


Figure 15. Tube Deformation Shapes at Various Times During Limit Cycle

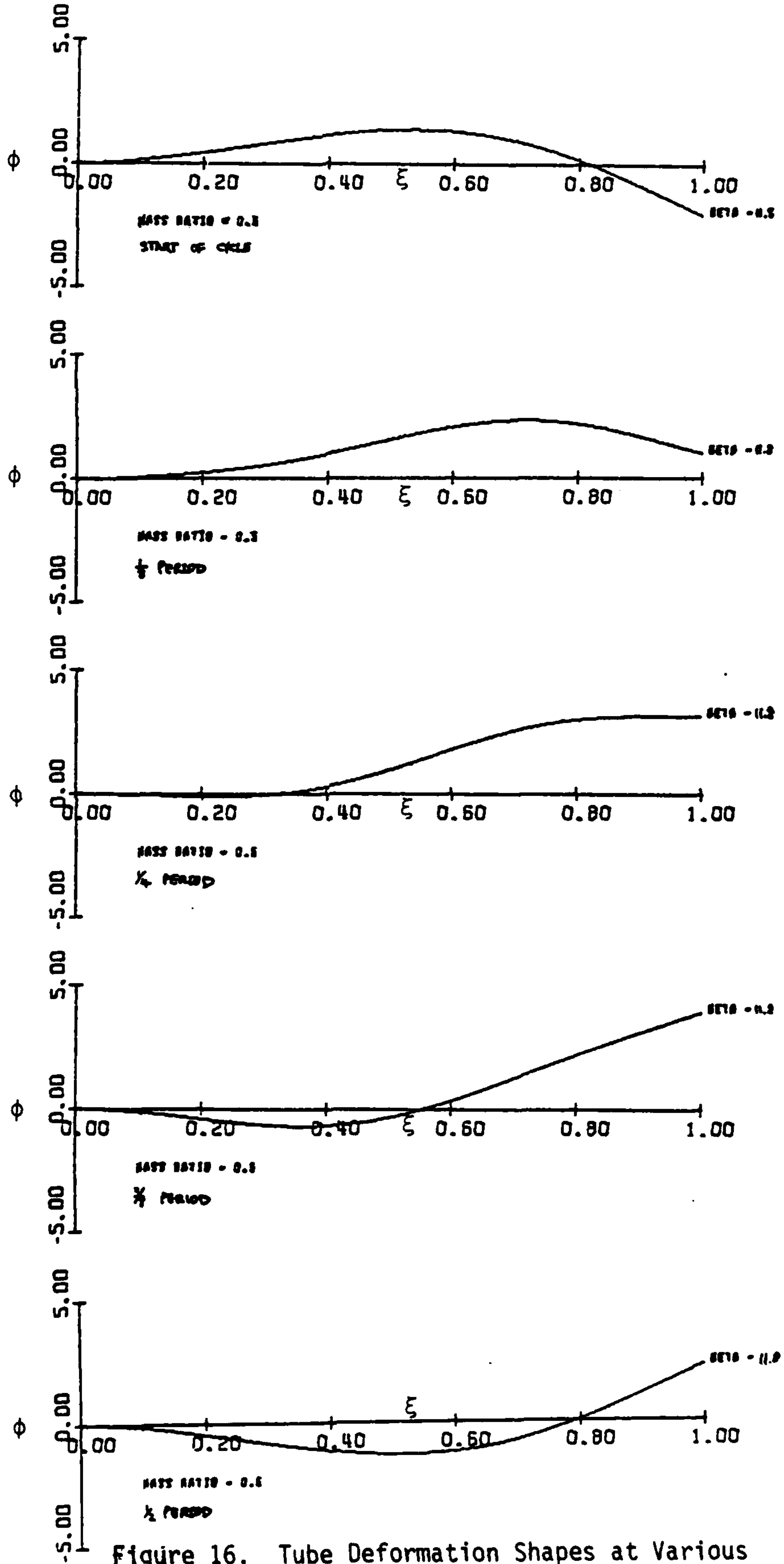


Figure 16. Tube Deformation Shapes at Various Times During Limit Cycle

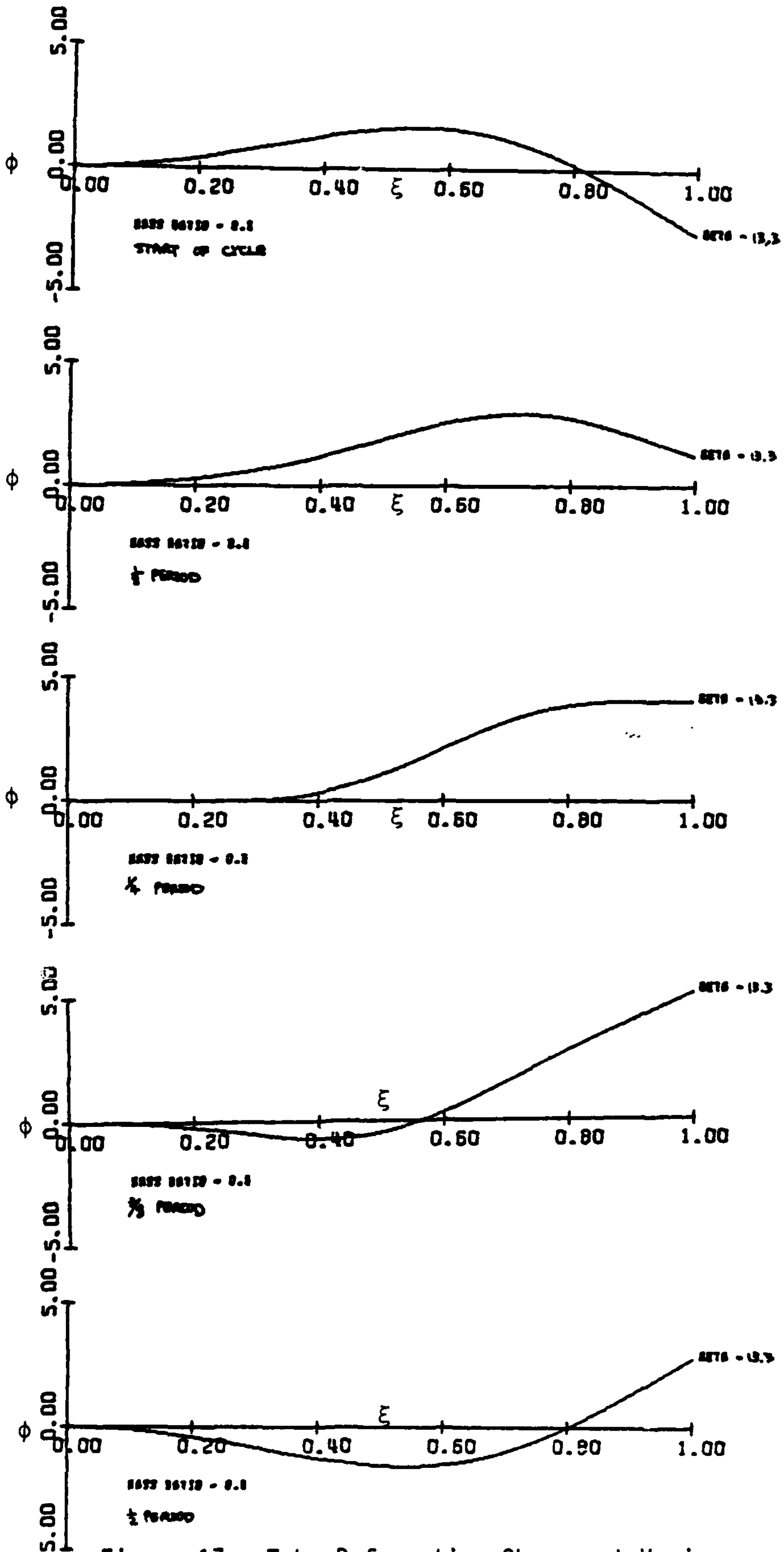


Figure 17. Tube Deformation Shapes at Various Times During Limit Cycle

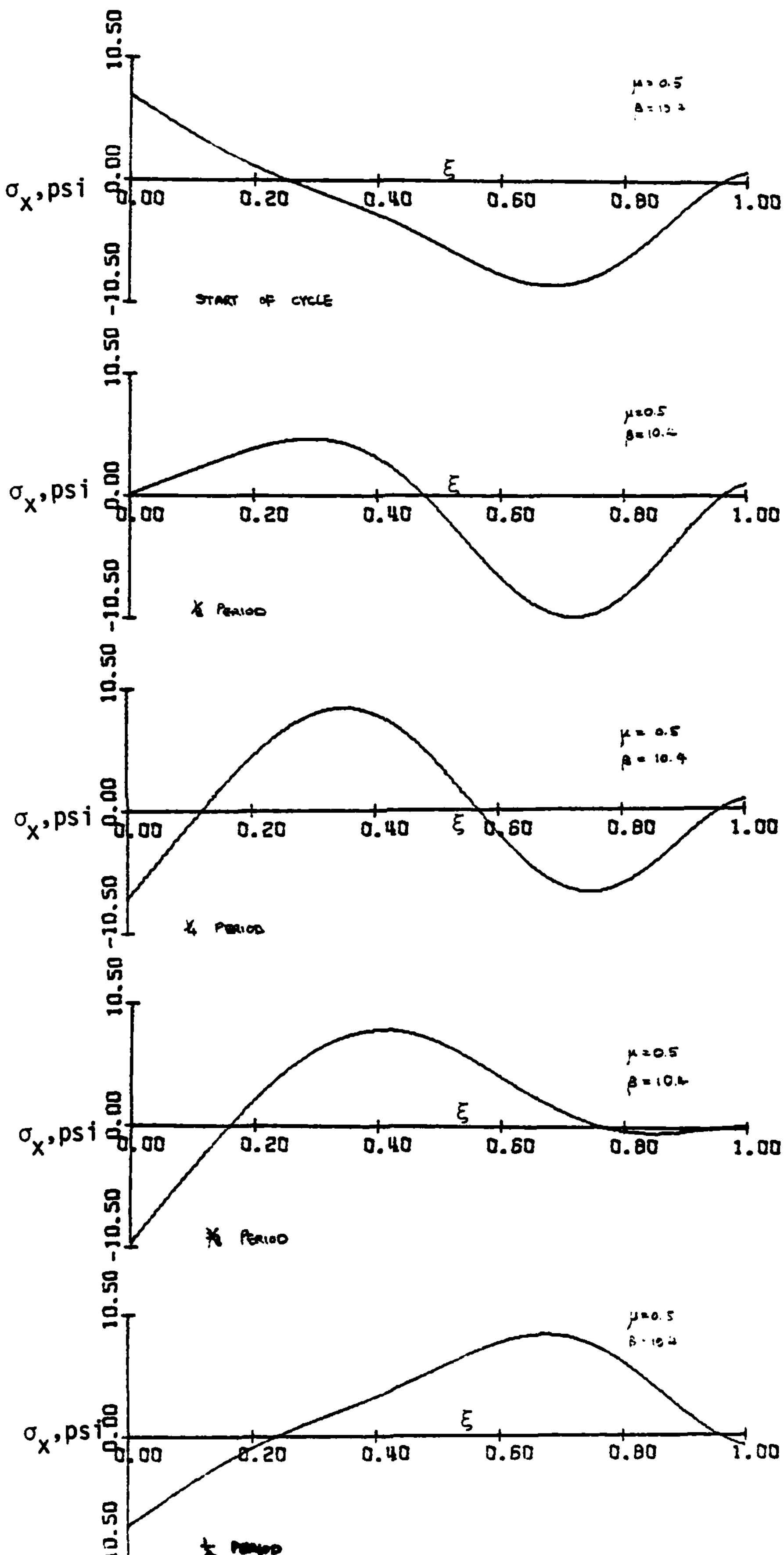


Figure 18. Tube Stress Distribution at Various Times During Limit Cycle

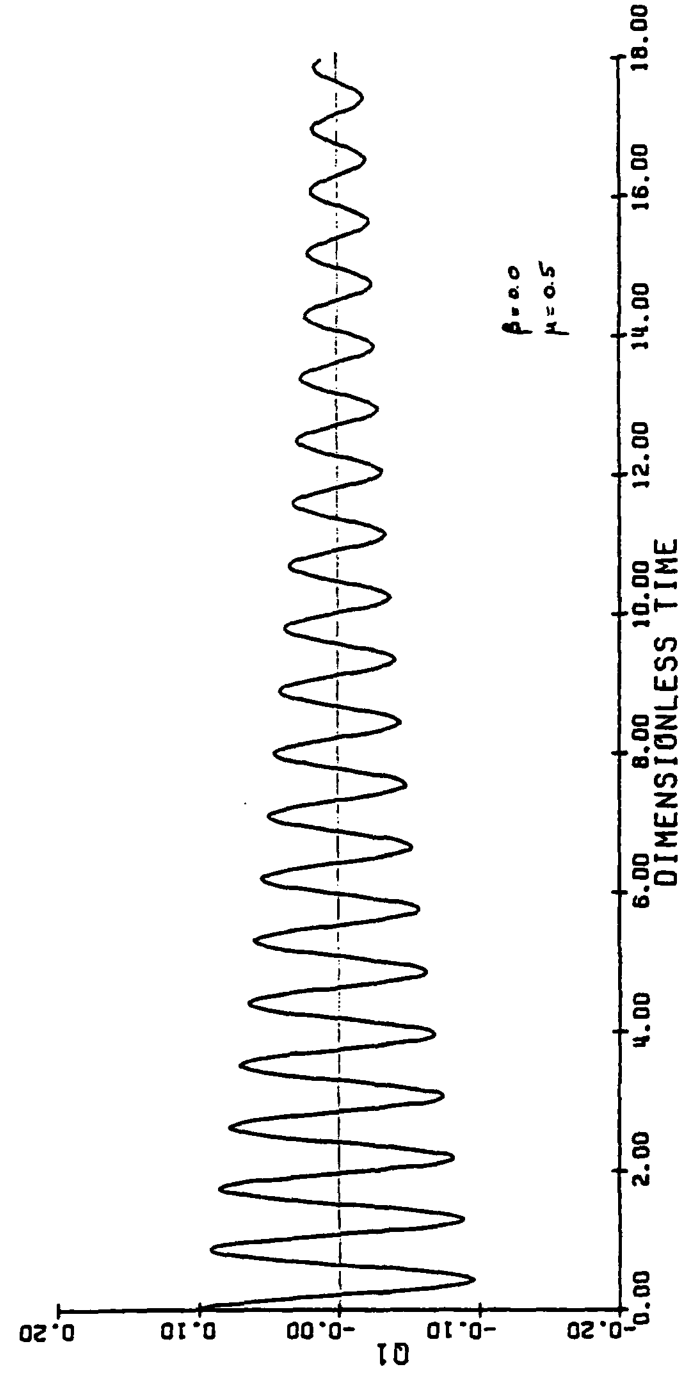


Figure 19a. First Mode Time History of Tube Motion
(Damped Vibration)

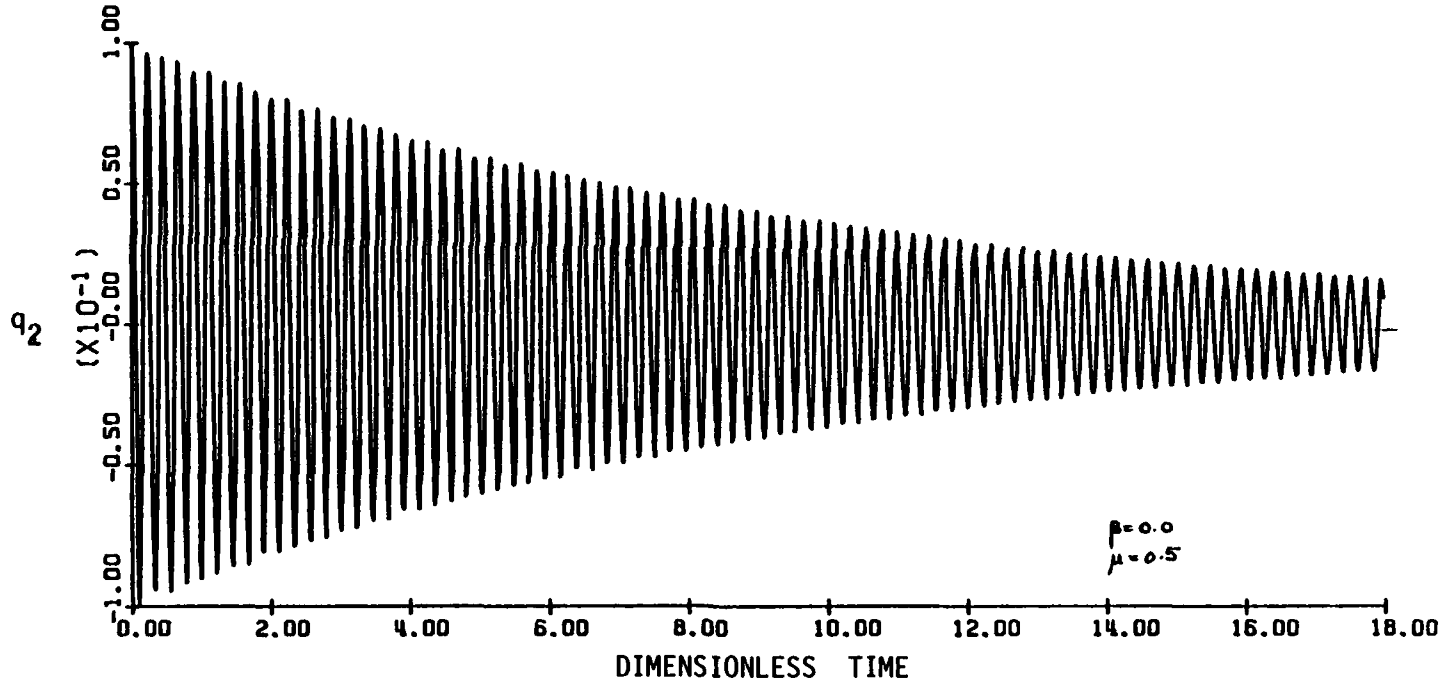


Figure 19b. Second Mode Time History of Tube Motion
(Damped Vibrations)

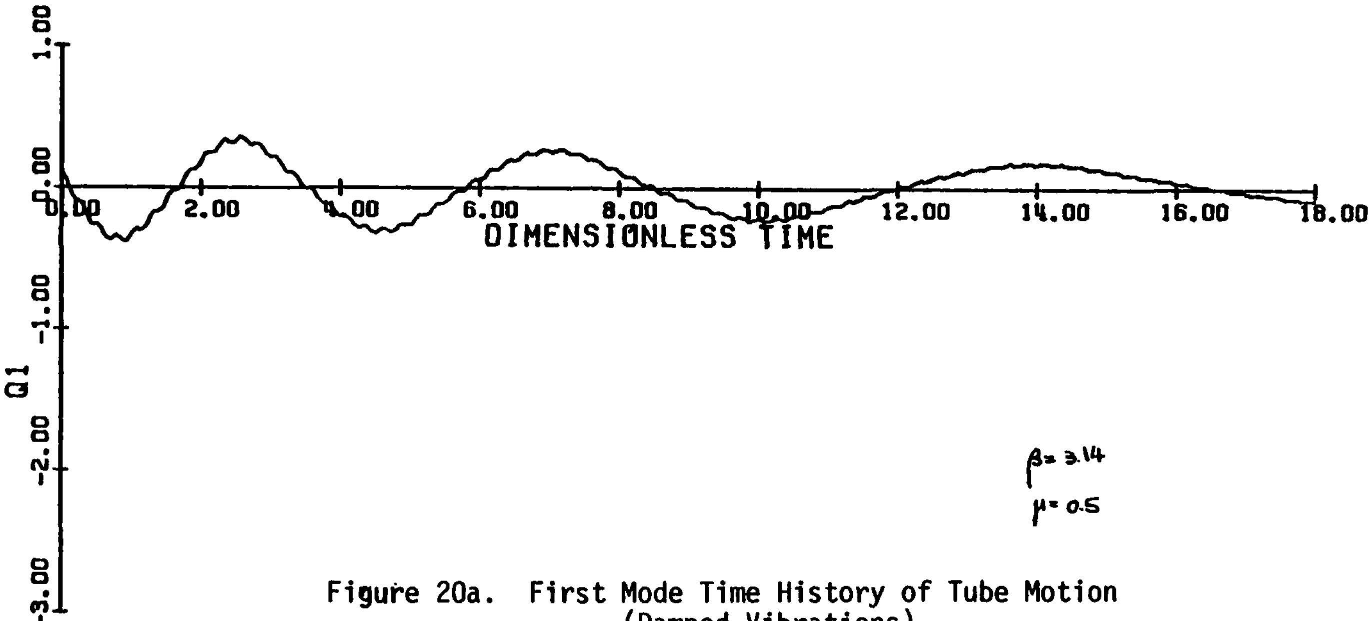


Figure 20a. First Mode Time History of Tube Motion (Damped Vibrations)

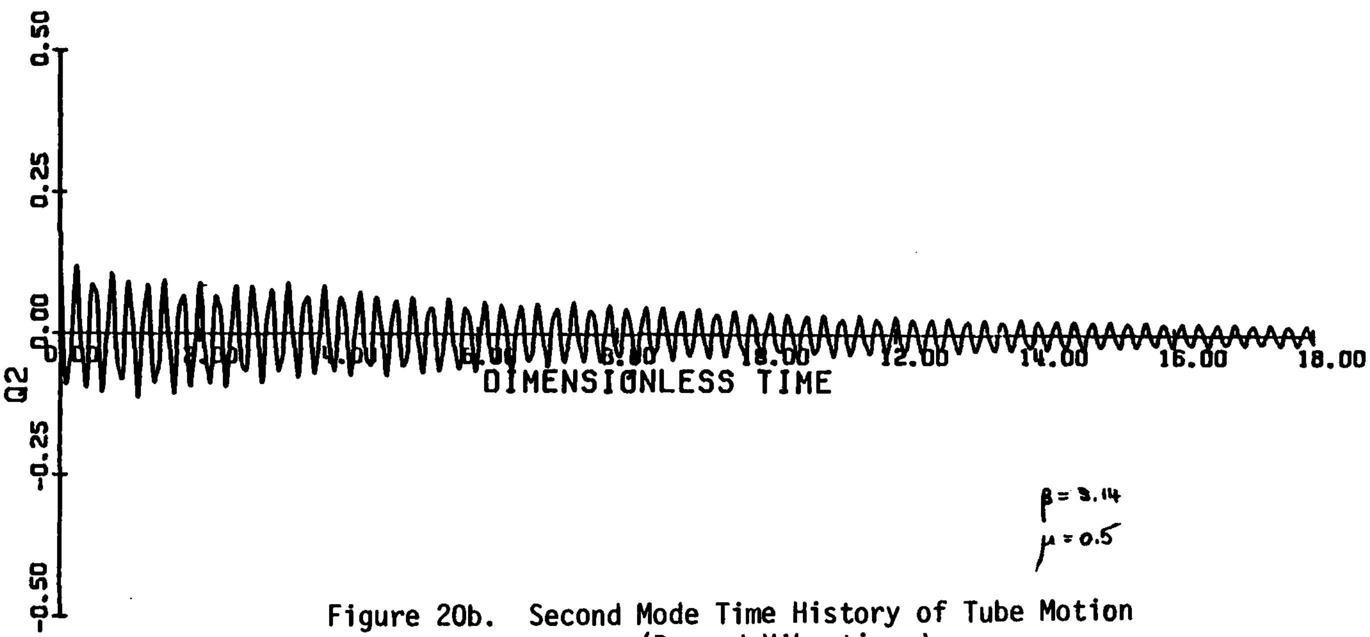


Figure 20b. Second Mode Time History of Tube Motion
(Damped Vibrations)

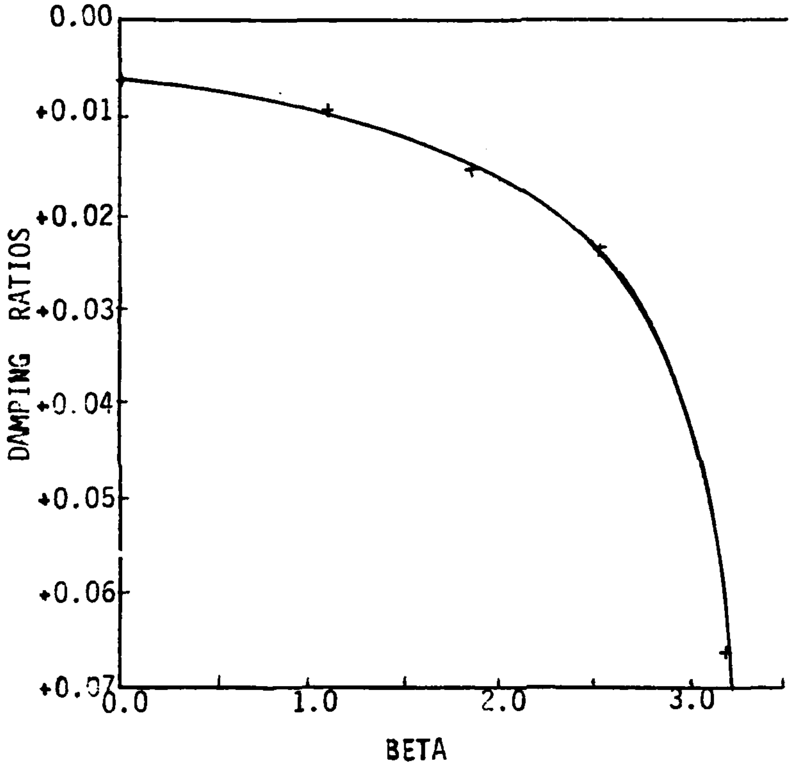


Figure 21a. Effects of Beta on First Mode Damping Ratios (μ = 0.1)

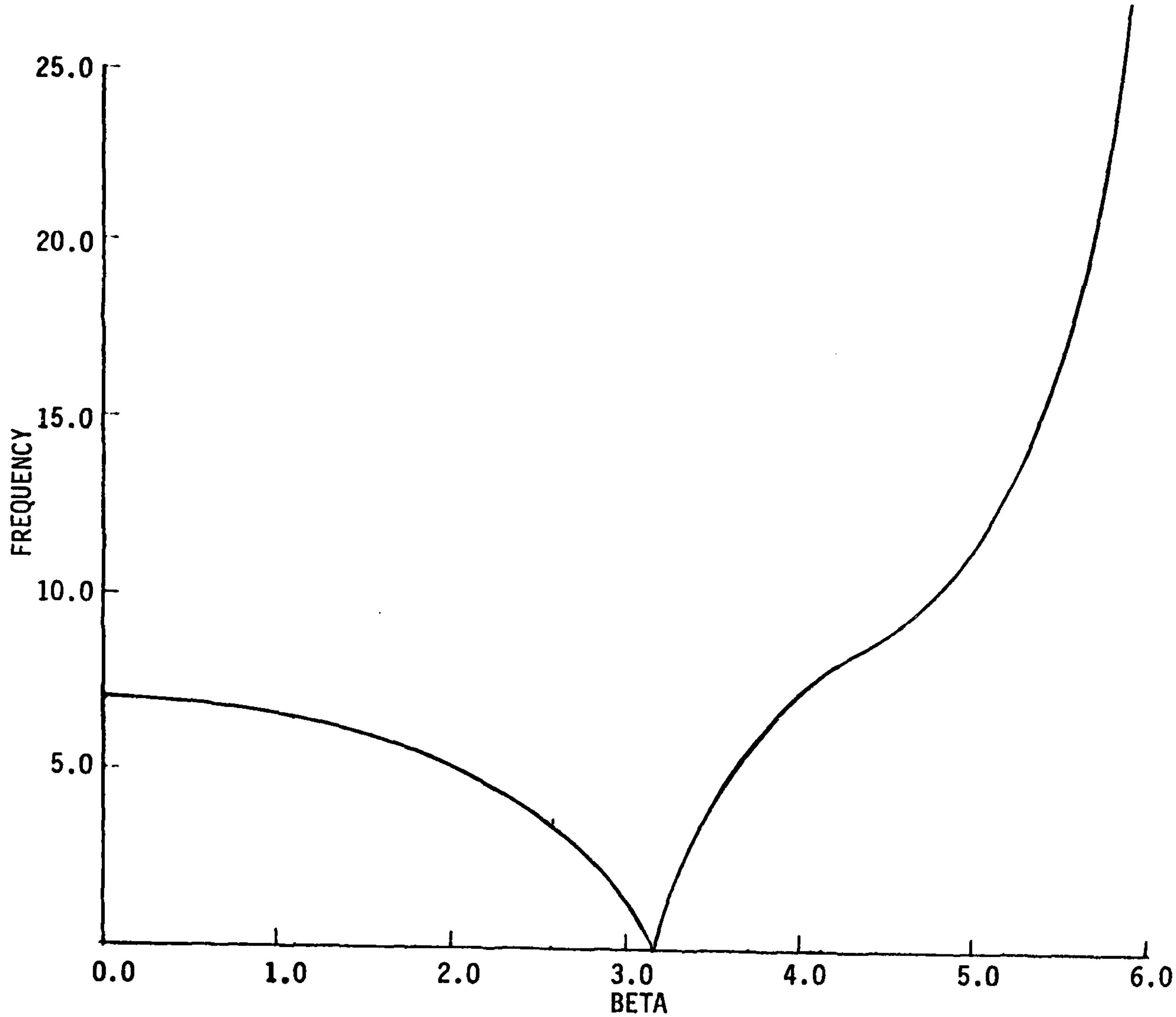


Figure 21b. Effects of Beta on First Mode Frequencies
($\mu = 0.1$)

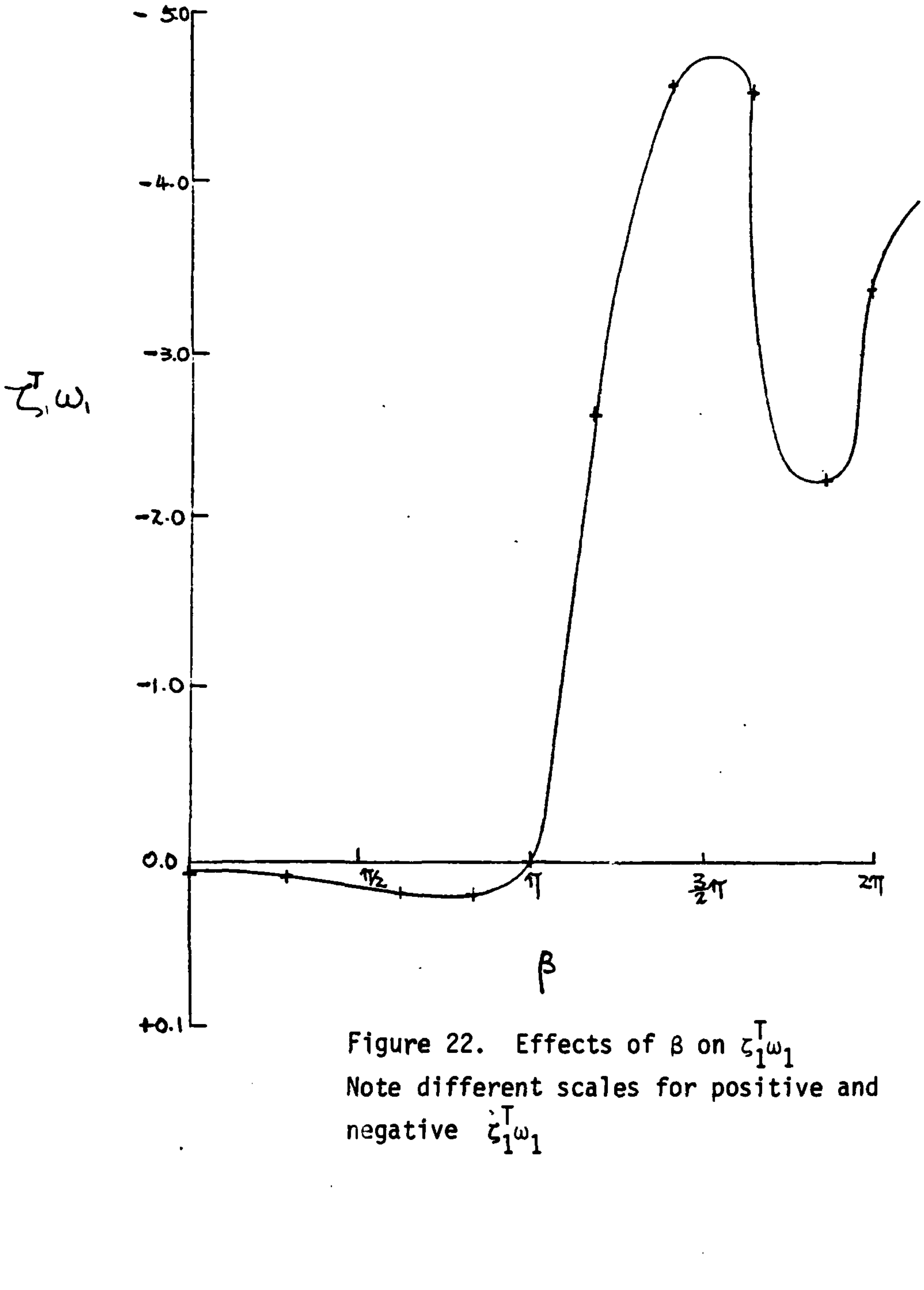


Figure 22. Effects of β on $\zeta_1^T\omega_1$
Note different scales for positive and negative $\zeta_1^T\omega_1$

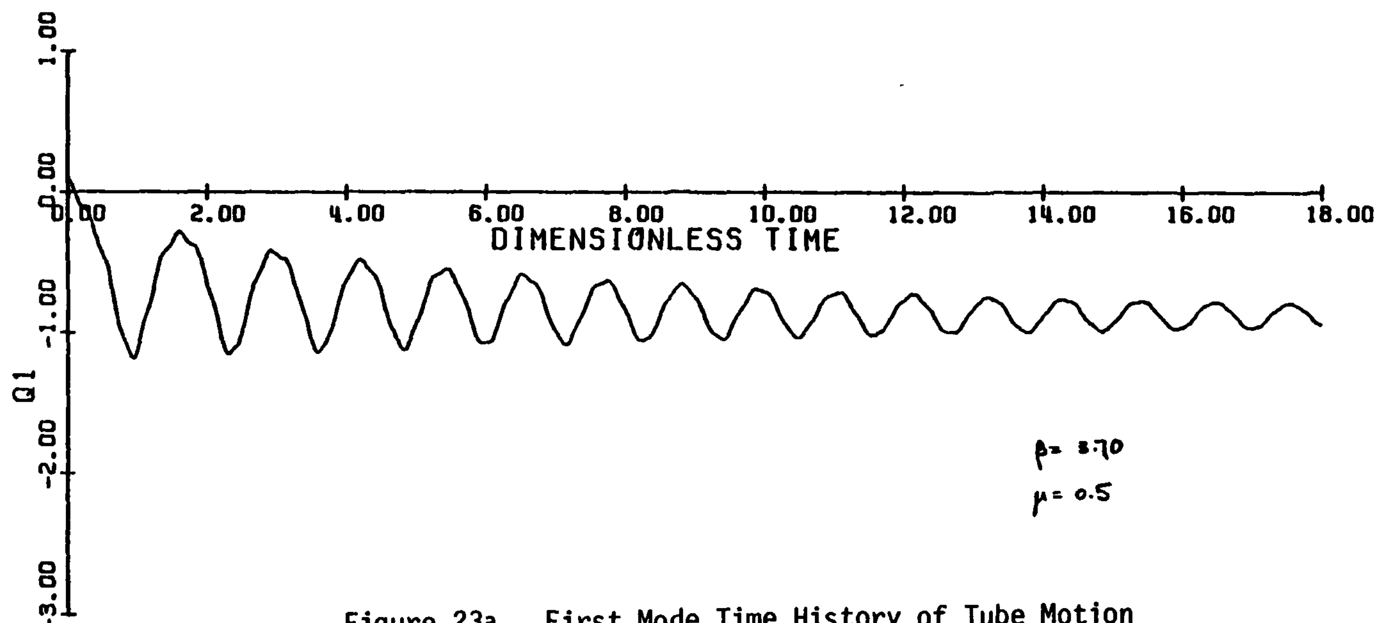


Figure 23a. First Mode Time History of Tube Motion
(Divergence)

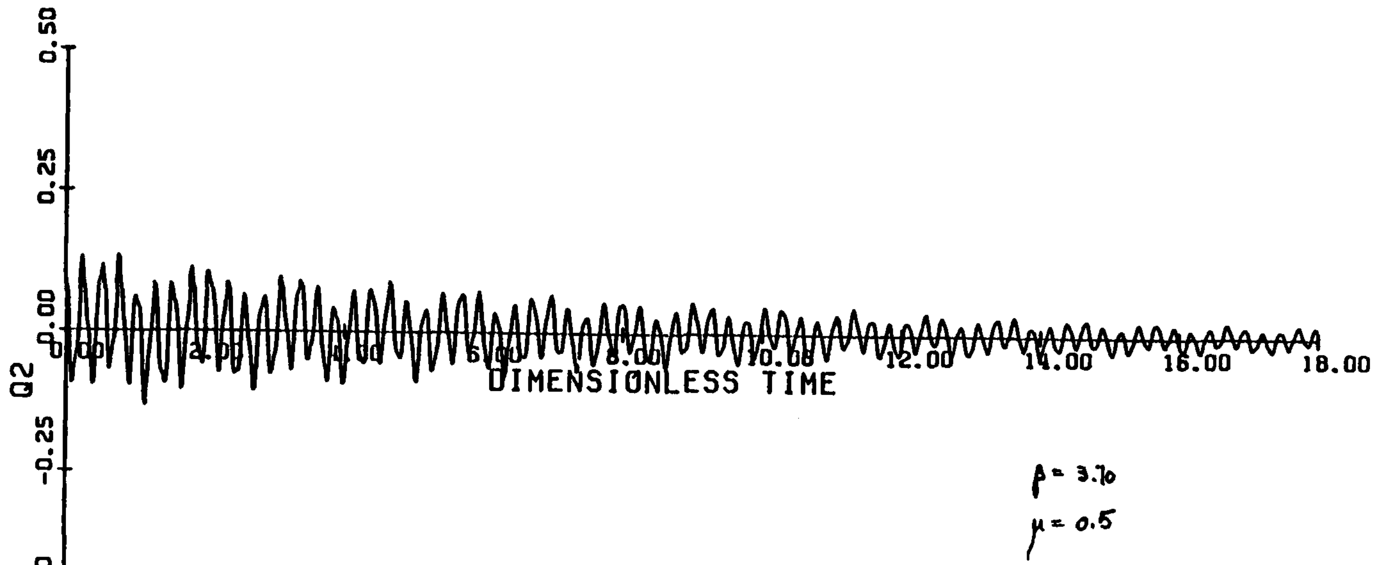


Figure 23b. Second Mode Time History of Tube Motion
(Divergence)

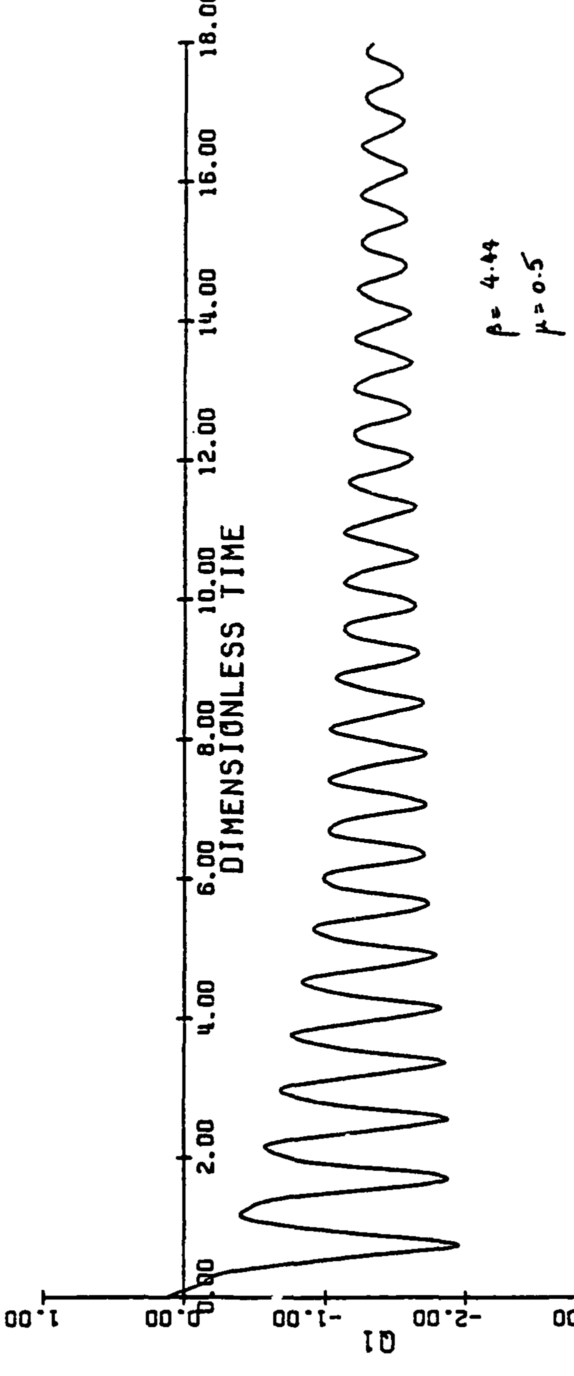


Figure 24a. First Mode Time History of Tube Motion (Divergence)

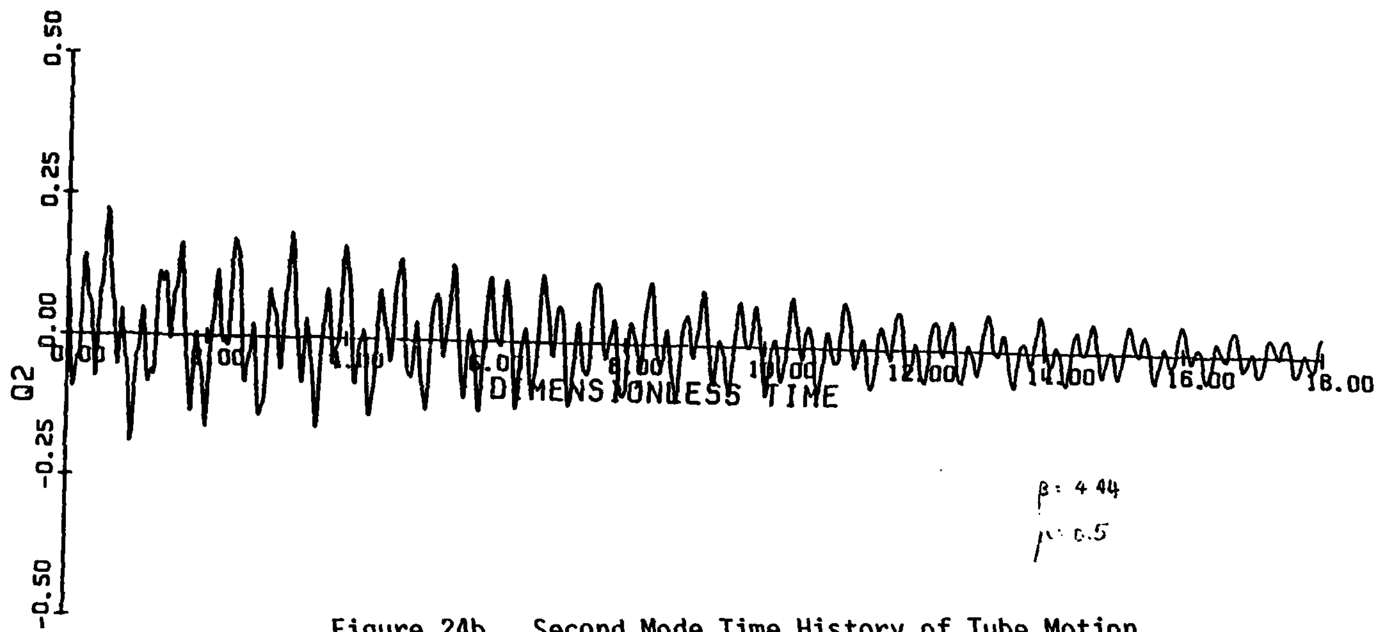


Figure 24b. Second Mode Time History of Tube Motion
(Divergence)

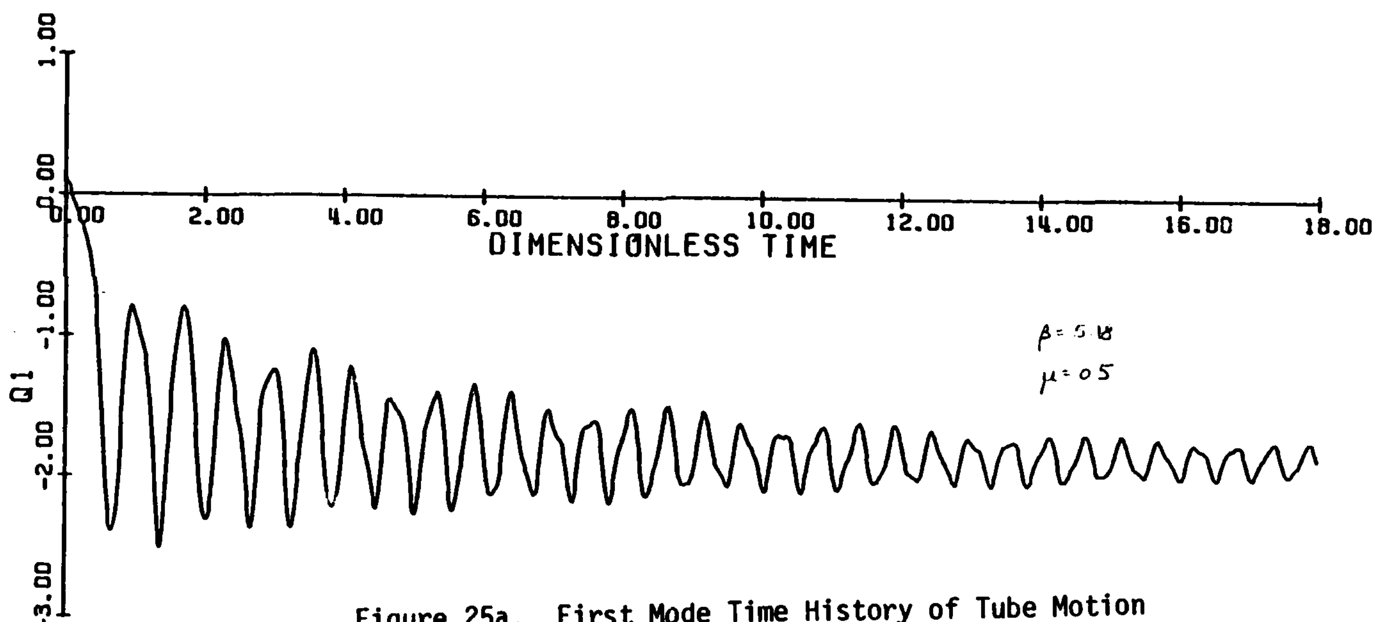


Figure 25a. First Mode Time History of Tube Motion
(Divergence)

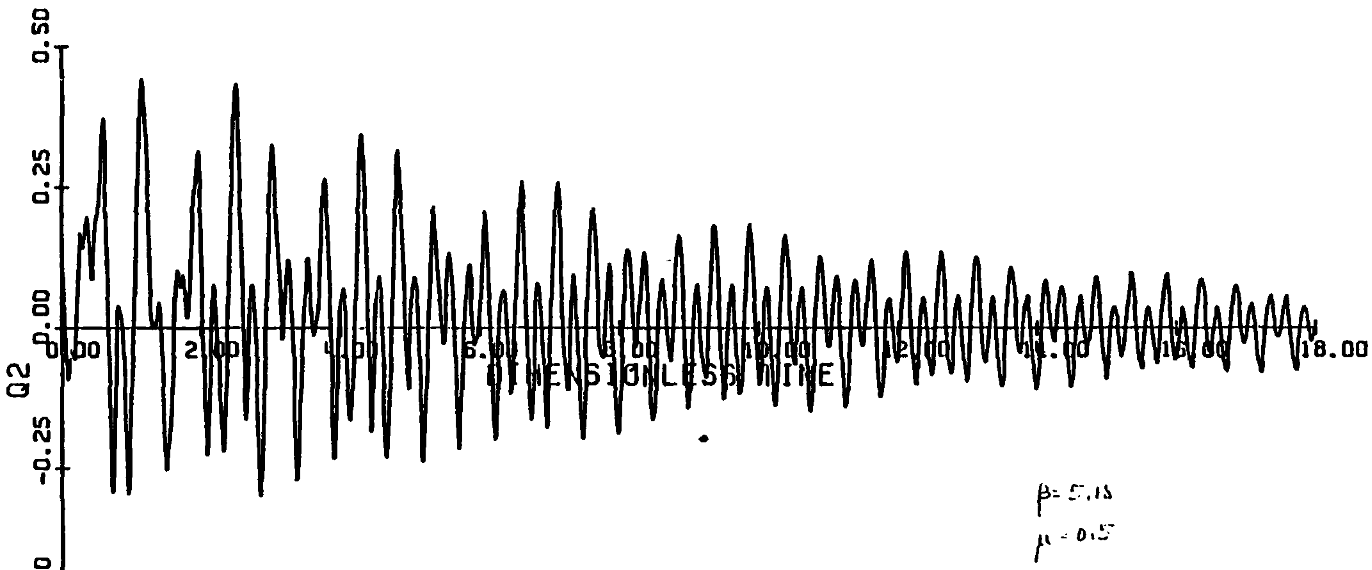


Figure 25b. Second Mode Time History of Tube Motion
(Divergence)

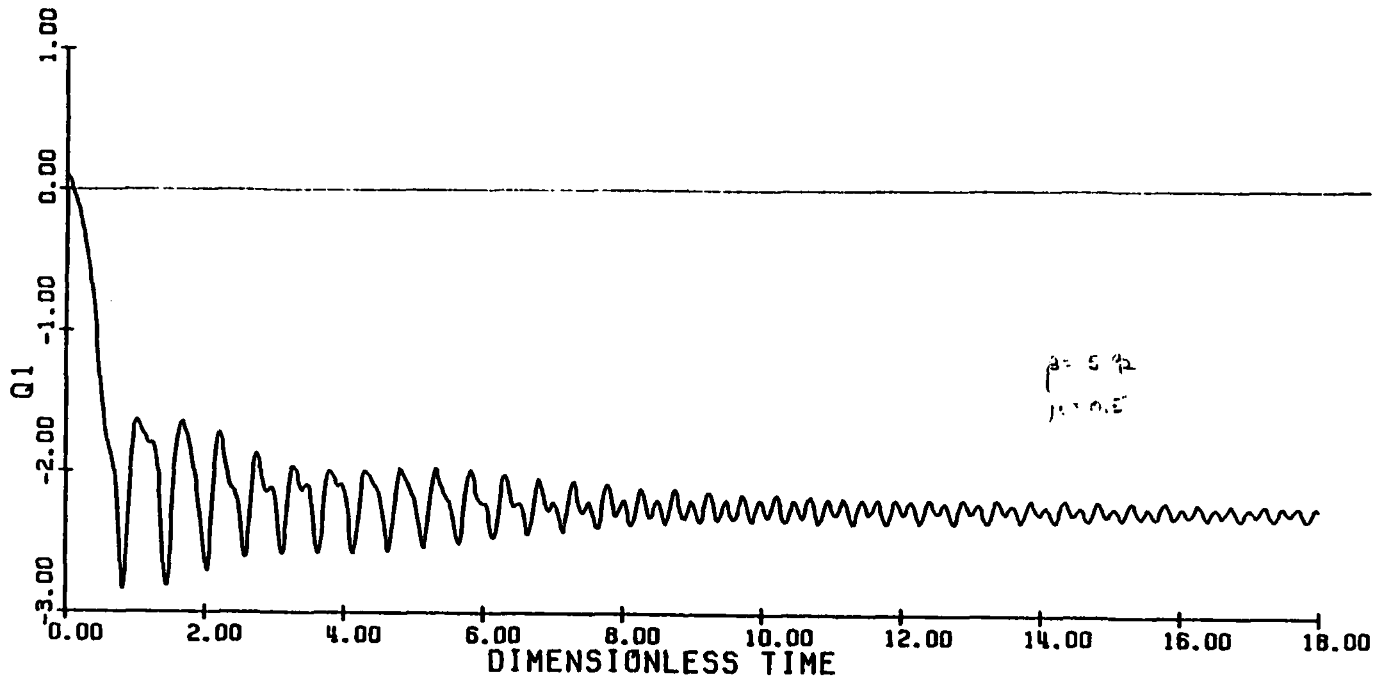


Figure 26a. First Mode Time History of Tube Motion
(Divergence)

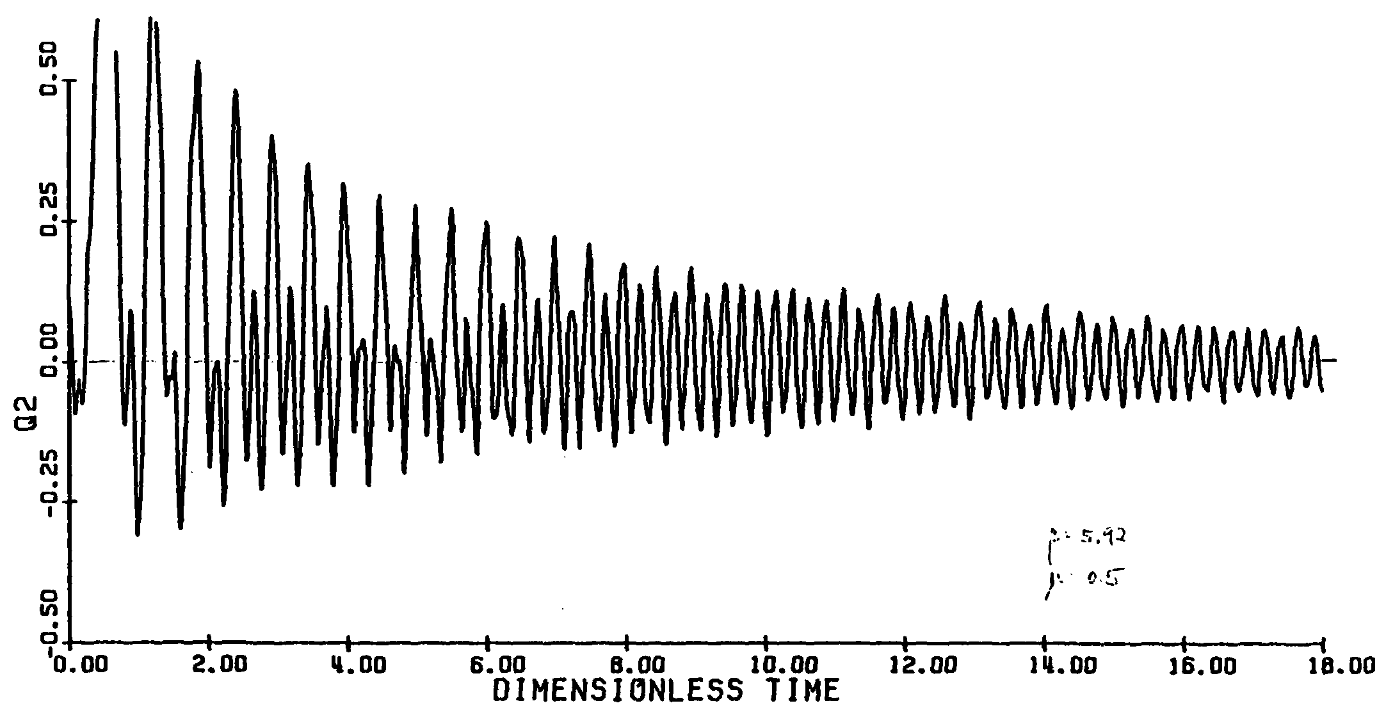


Figure 26b. Second Mode Time History of Tube Motion
(Divergence)

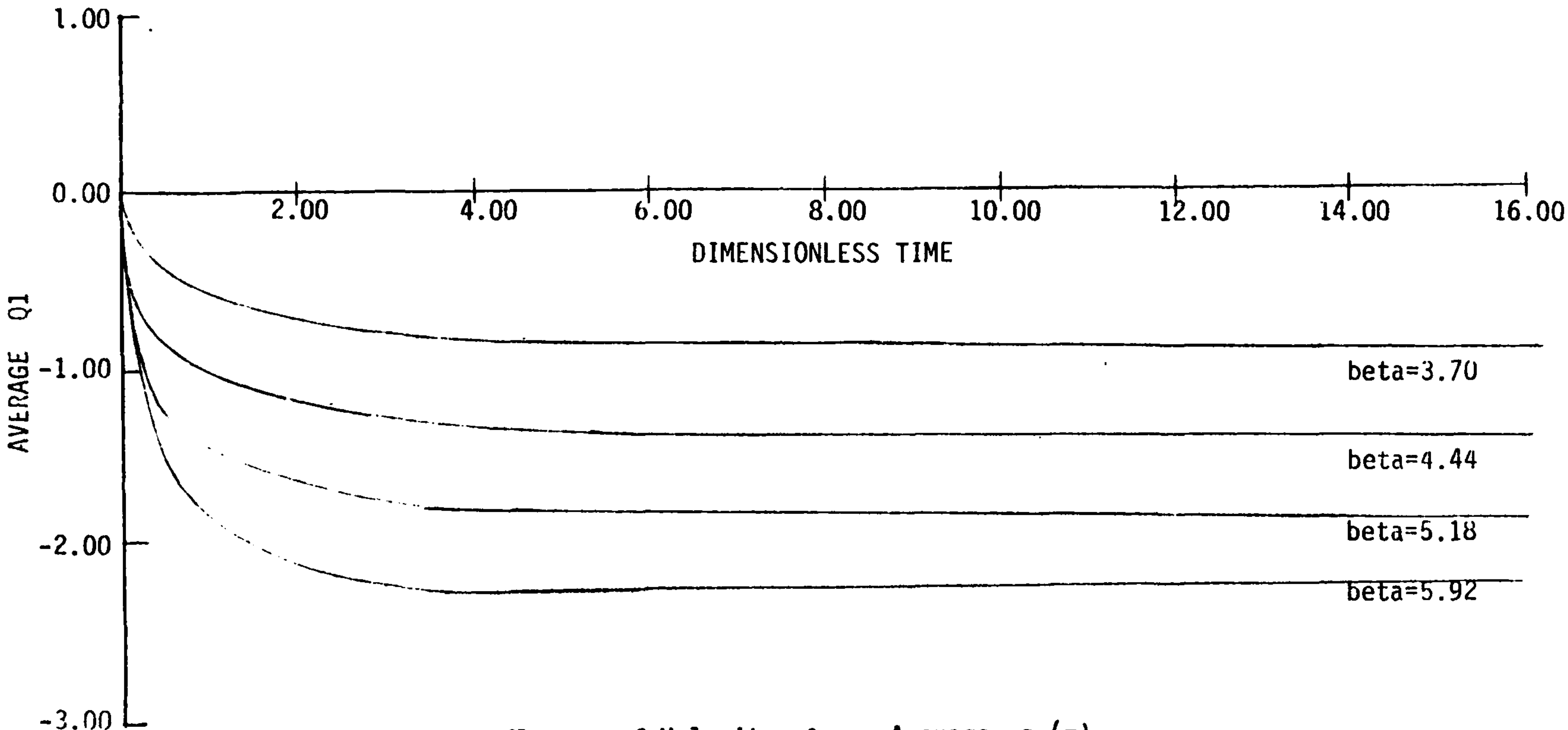


Figure 27. Influence of Velocity, β, on Average $q_1(\tau)$

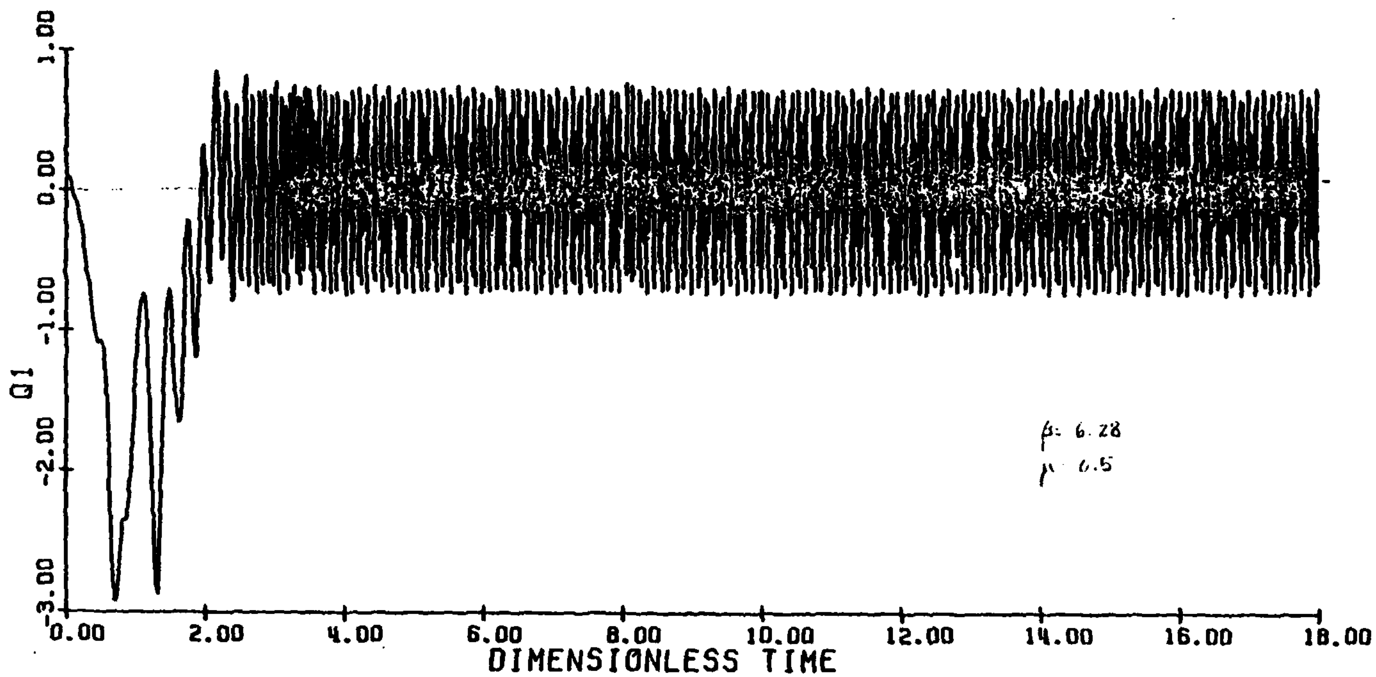


Figure 28a. Dynamic Instability of a Simply Supported Tube
(First Mode)

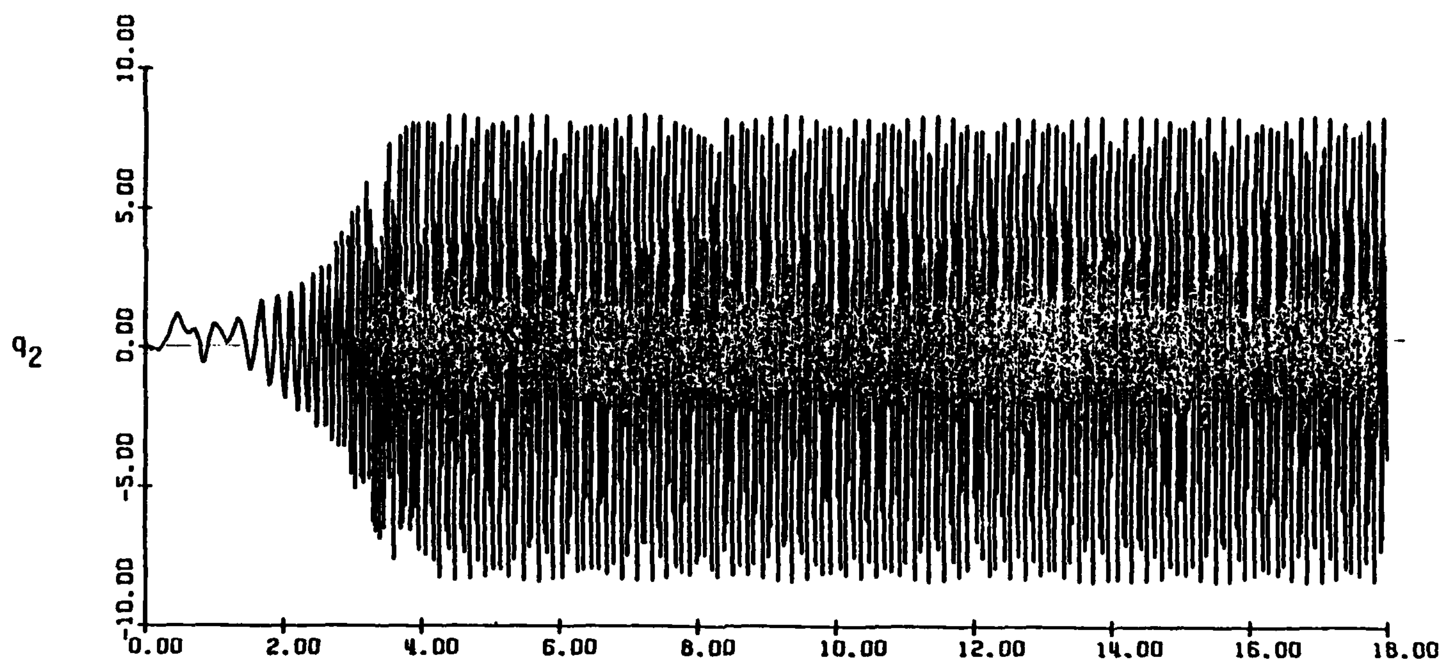


Figure 28b. Dynamic Instability of a Simply Supported Tube
(Second Mode)

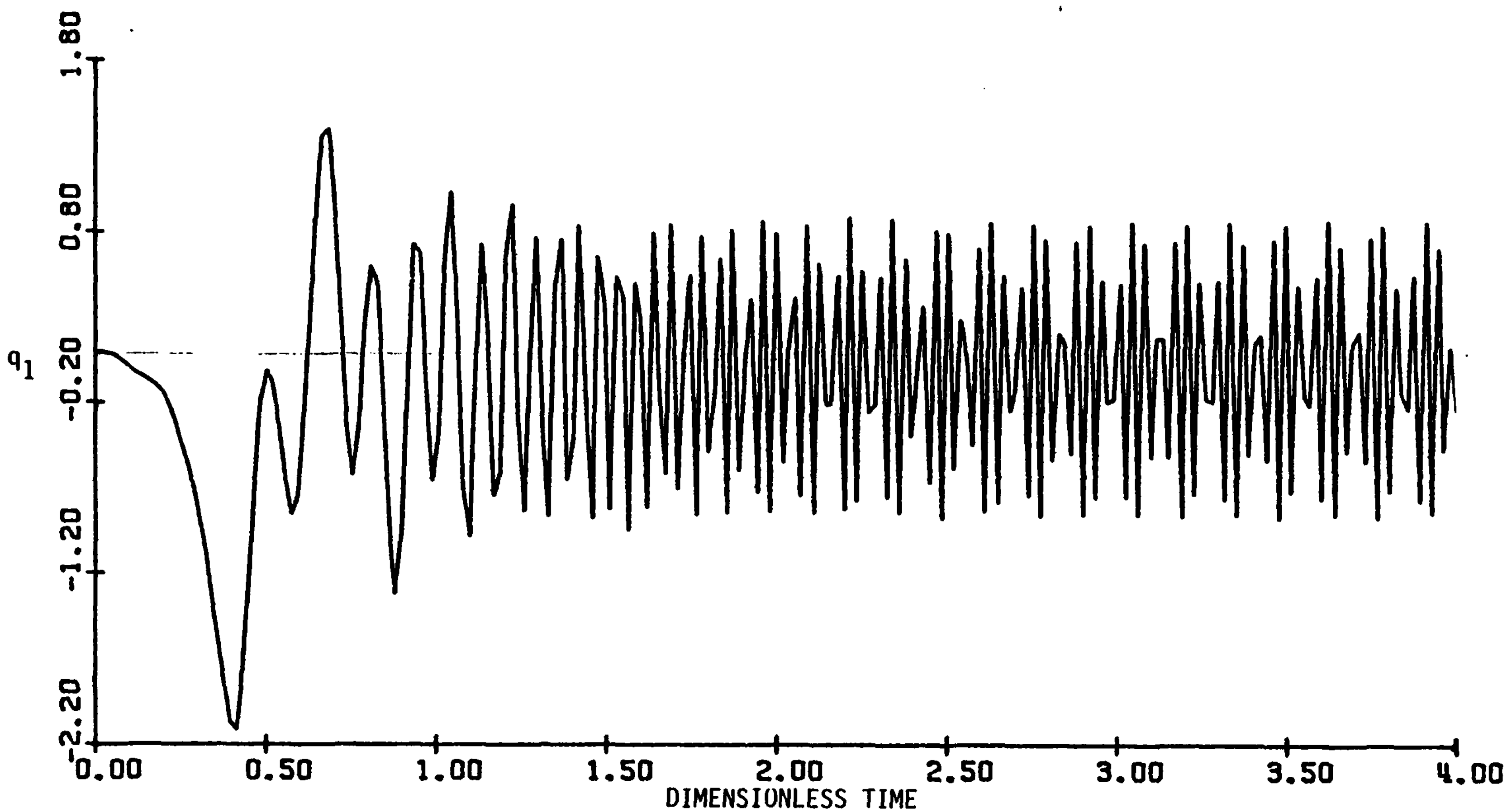


Figure 29a. First Mode Instability
(μ = 0.5, β = 7.4)

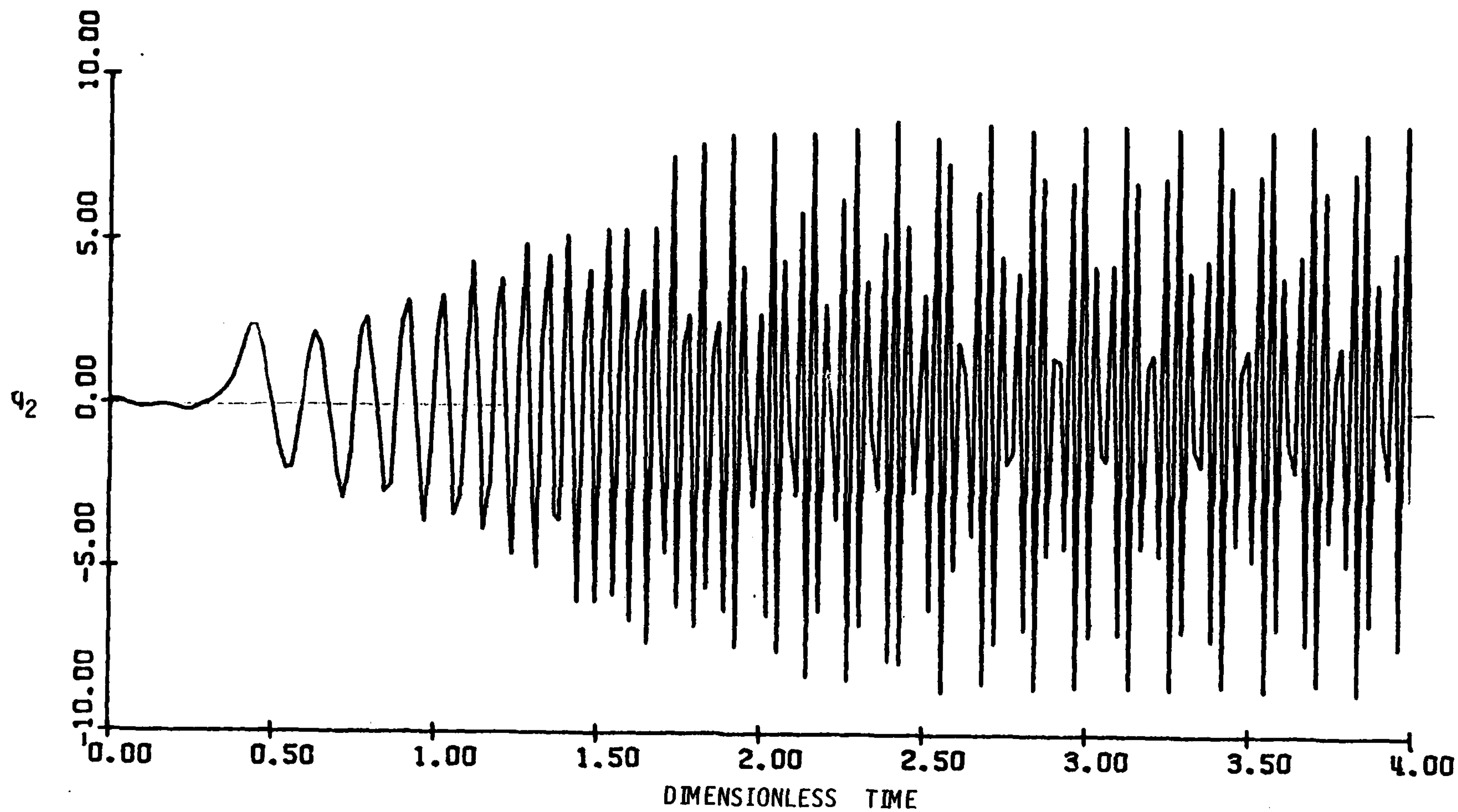


Figure 29b. Second Mode Instability
(μ = 0.5, β = 7.4)

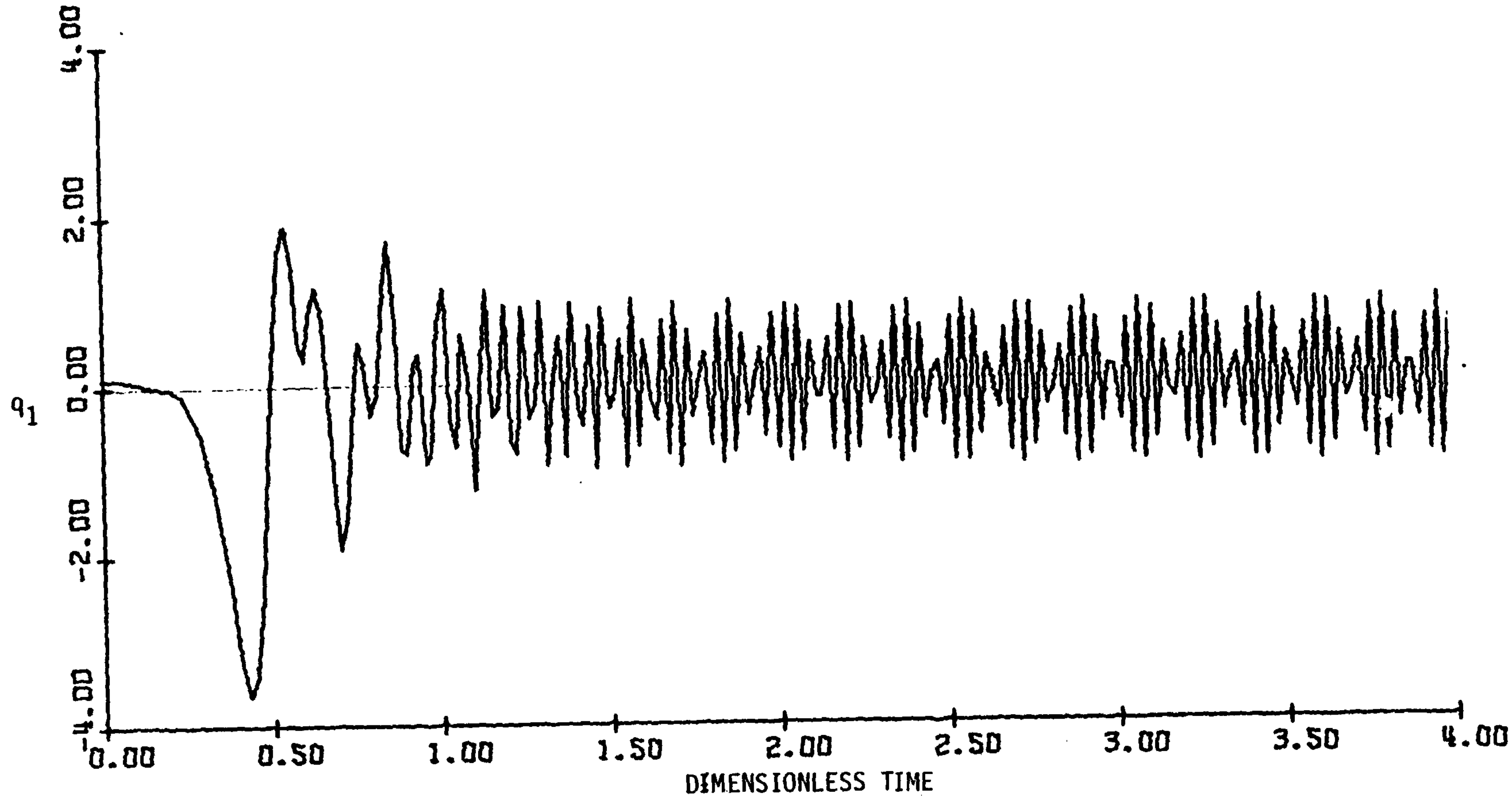


Figure 30a. First Mode Instability
($\mu = 0.5$, $\beta = 8.14$)

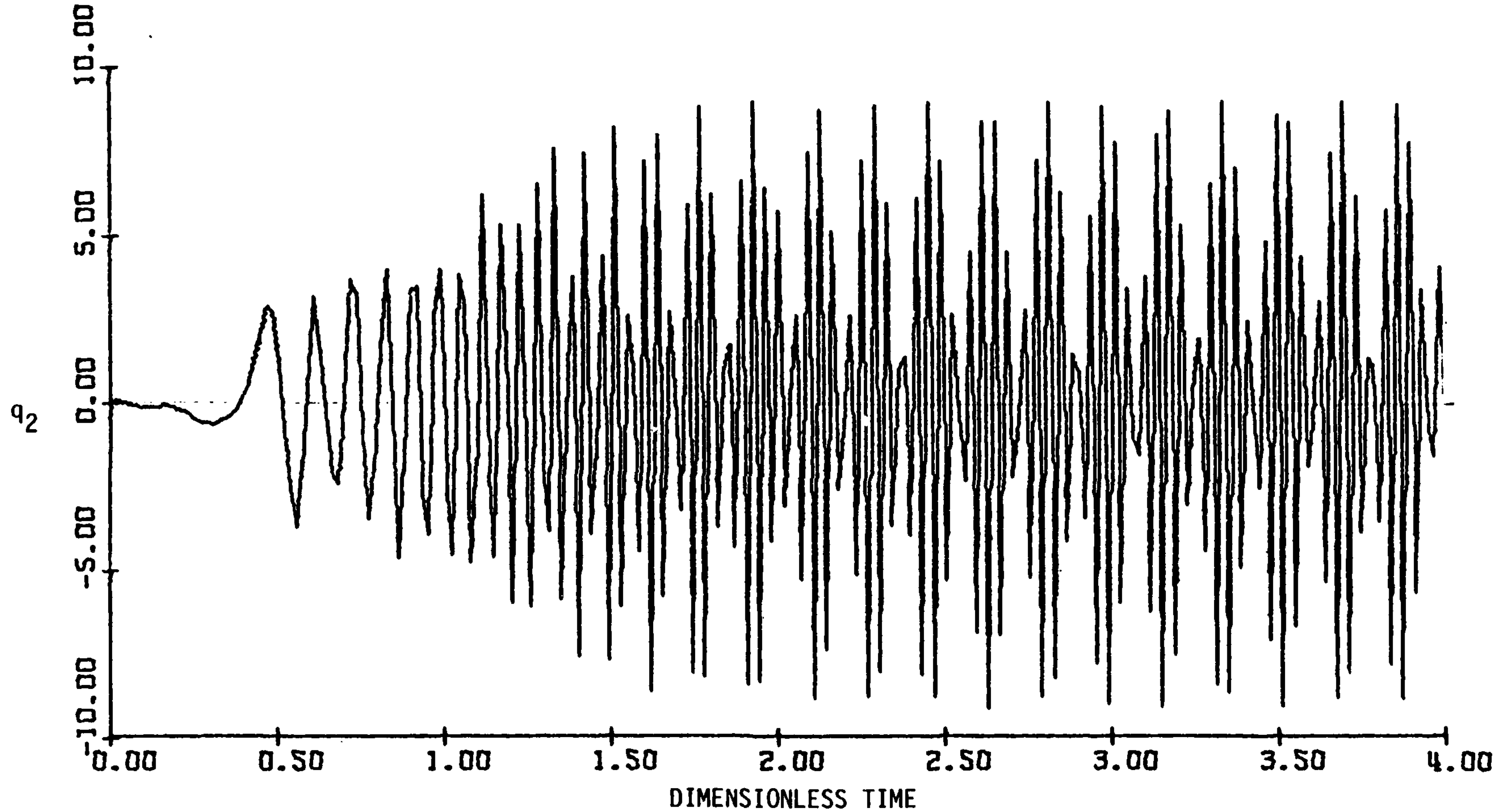


Figure 30b. Second Mode Instability
($\mu = 0.5$, $\beta = 8.14$)

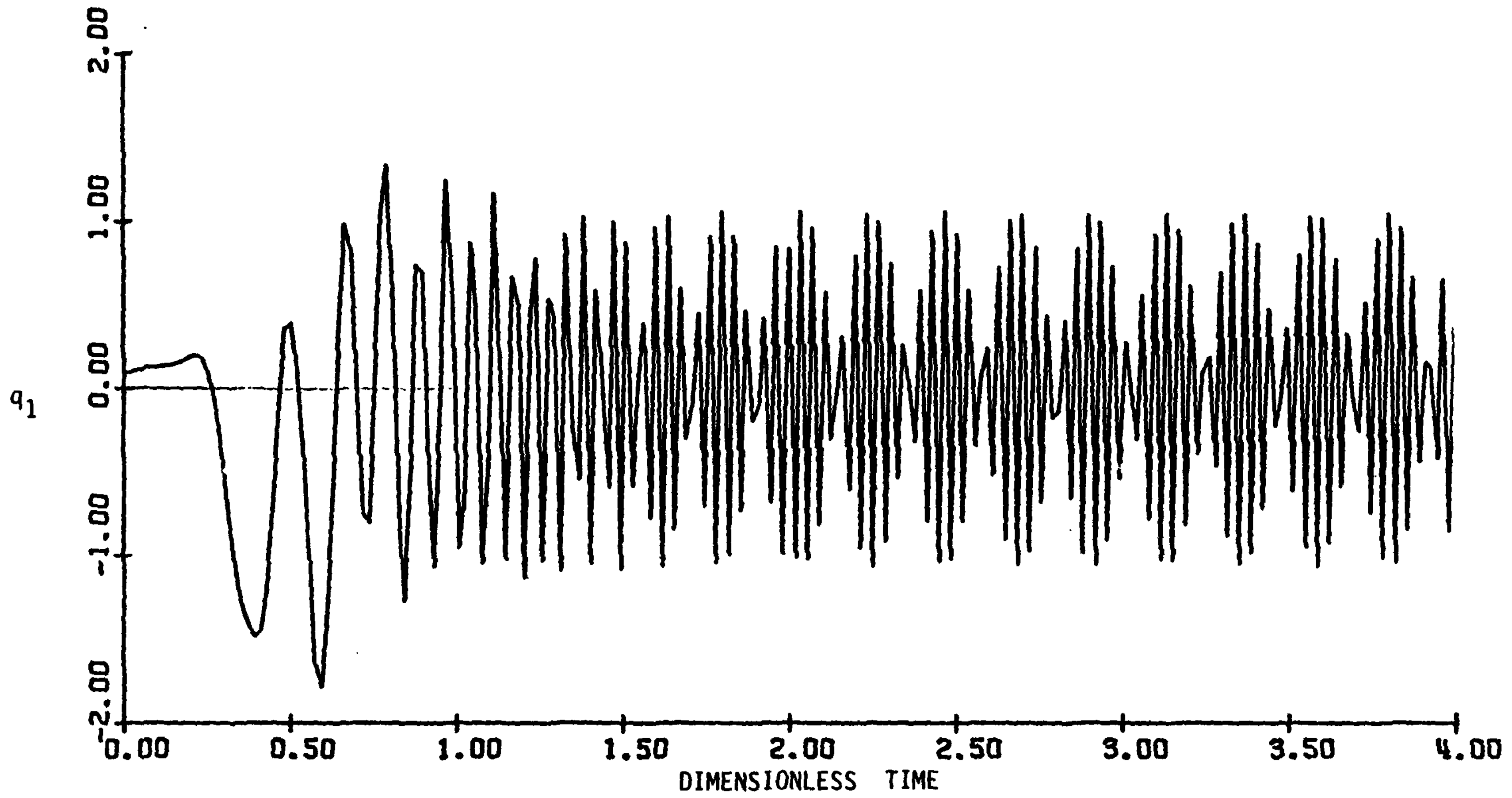


Figure 31a. First Mode Instability
($\mu = 0.5$, $\beta = 8.88$)

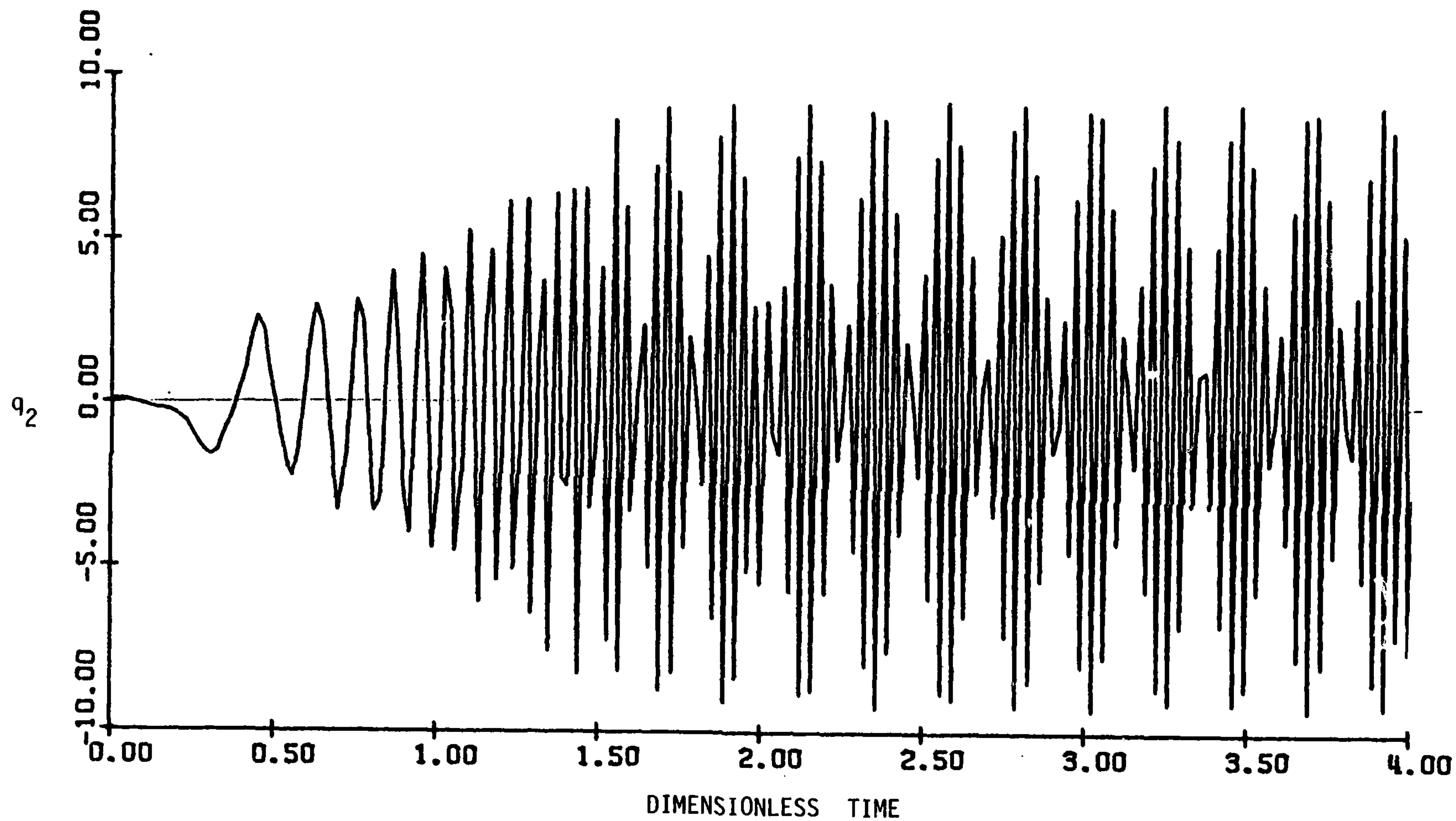


Figure 31b. Second Mode Instability
($\mu = 0.5$, $\beta = 8.88$)

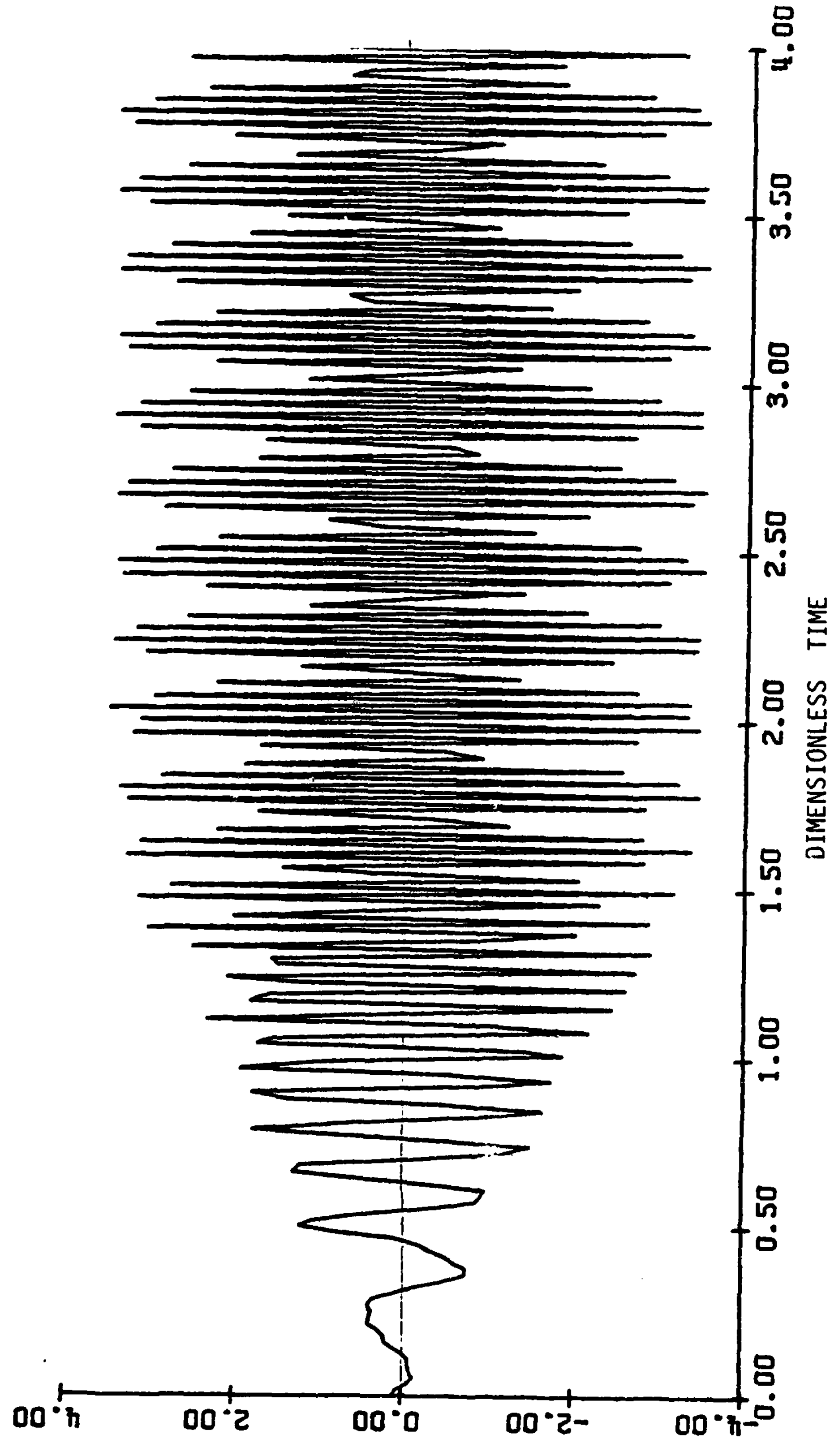


Figure 31c. *Third Mode Instability*
(μ = 0.5, β = 8.88)

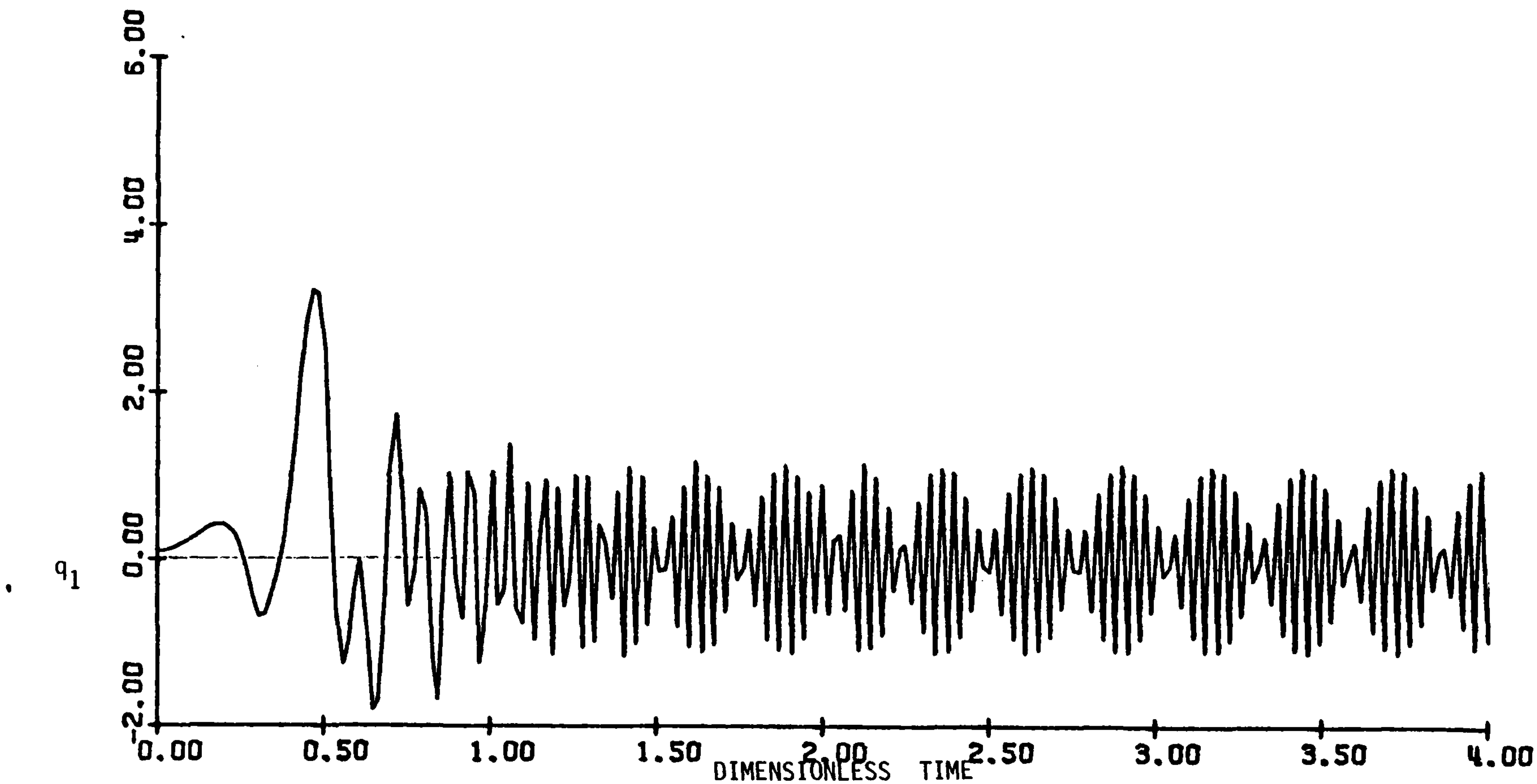


Figure 32a. First Mode Instability
(μ = 0.5, β = 9.42)

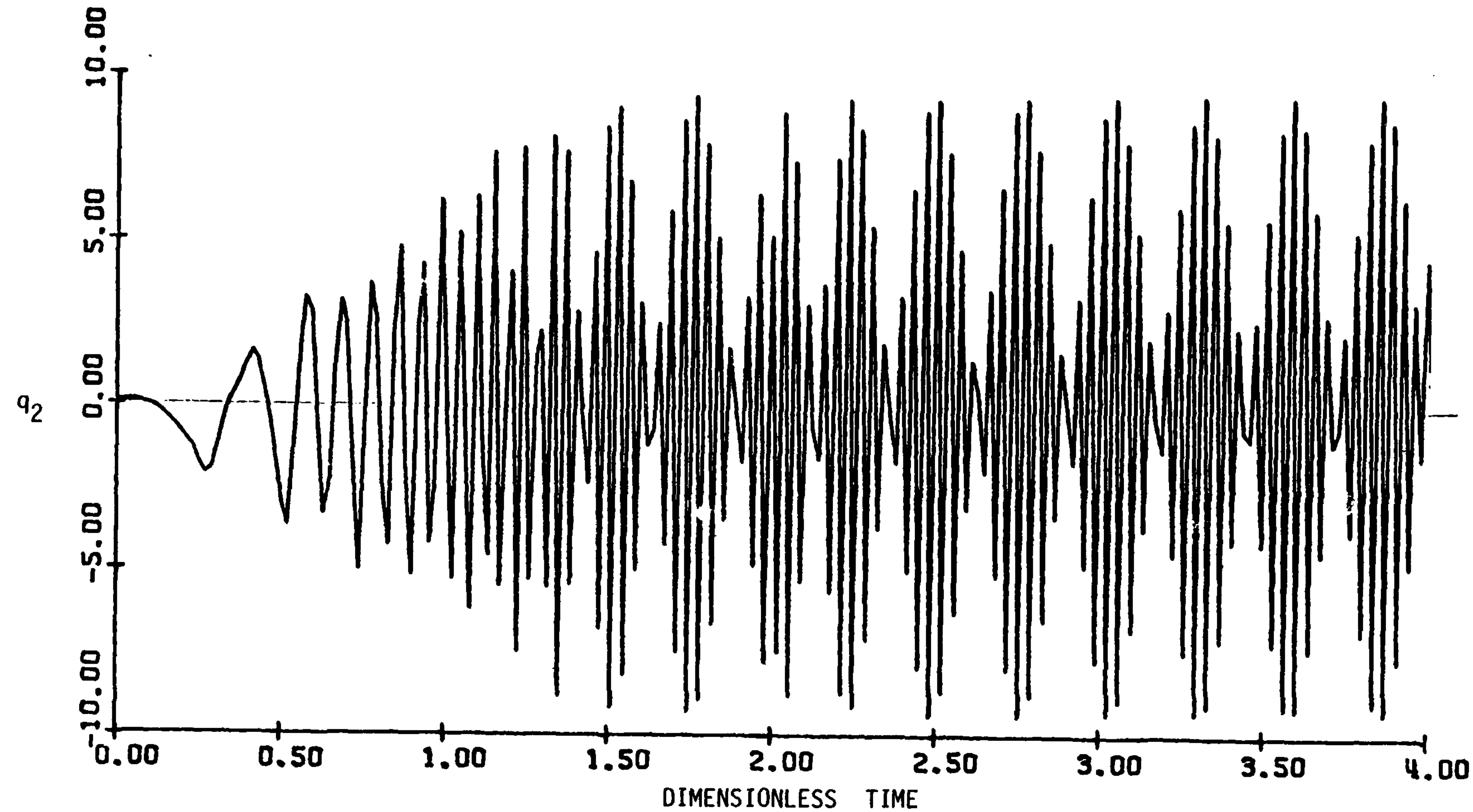


Figure 32b. Second Mode Instability
($\mu = 0.5$, $\beta = 9.42$)

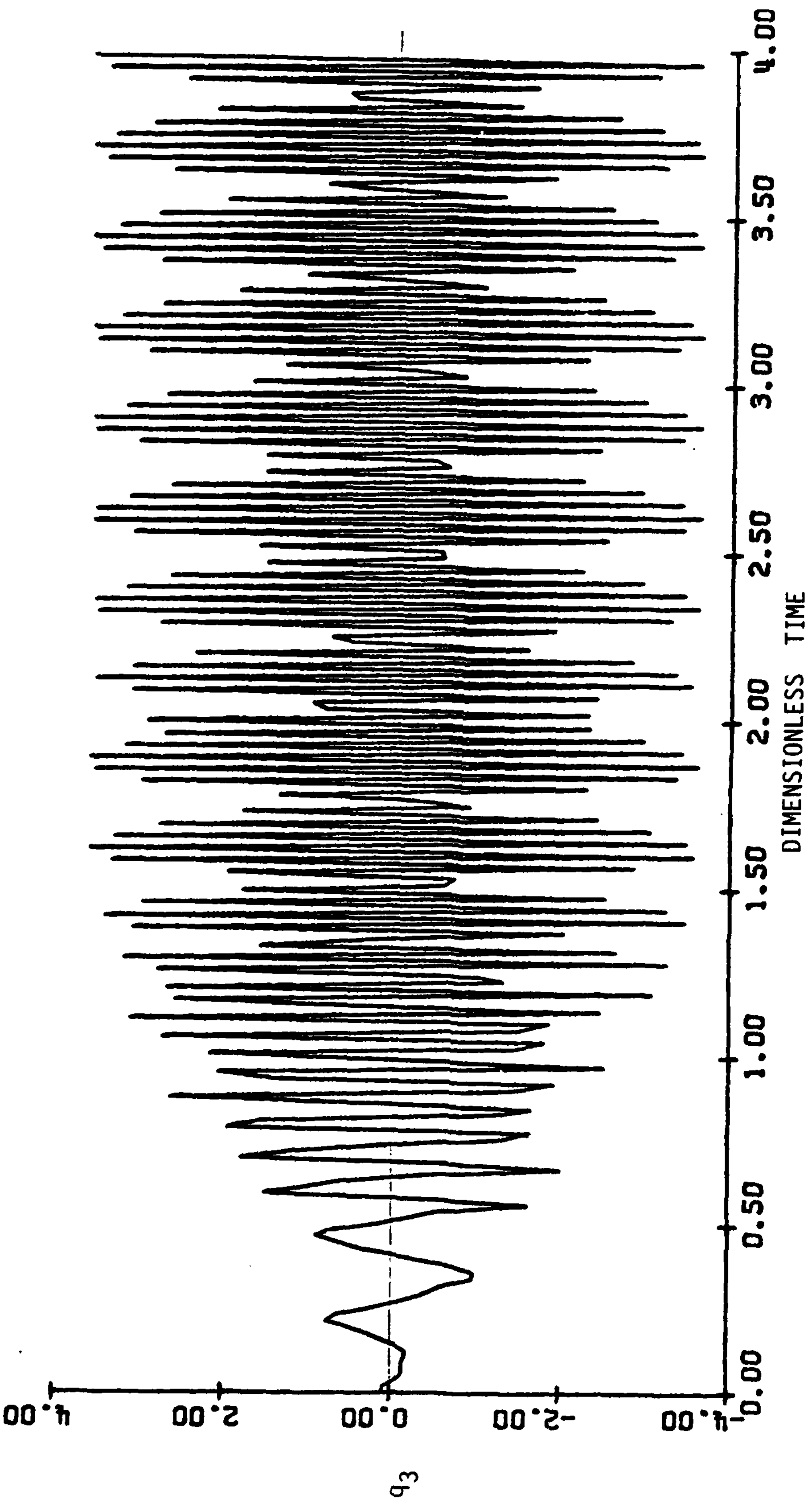


Figure 32c. Third Mode Instability
($\mu = 0.5$, $\beta = 9.42$)

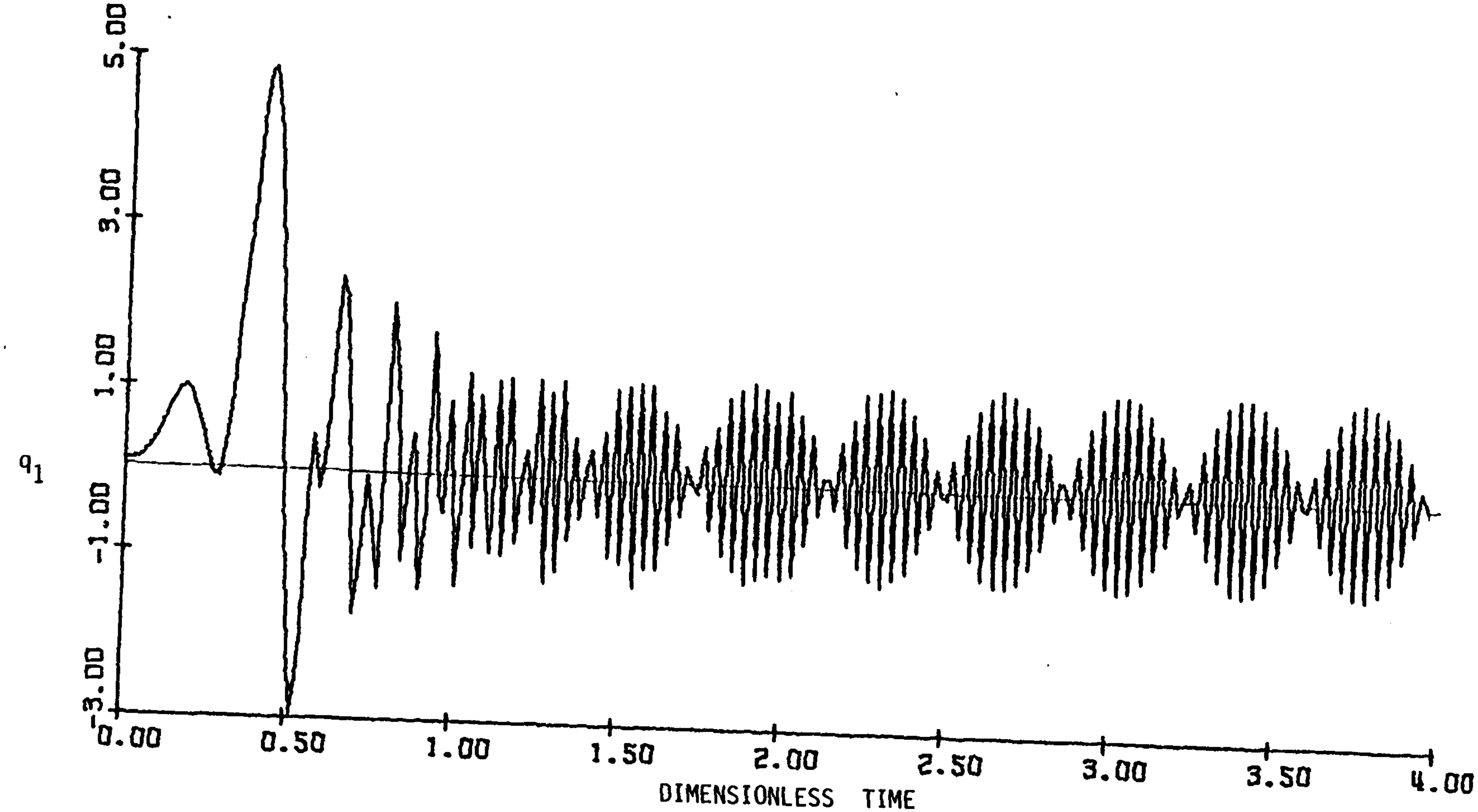


Figure 33a. First Mode Instability
($\mu = 0.5$, $\beta = 10.0$)

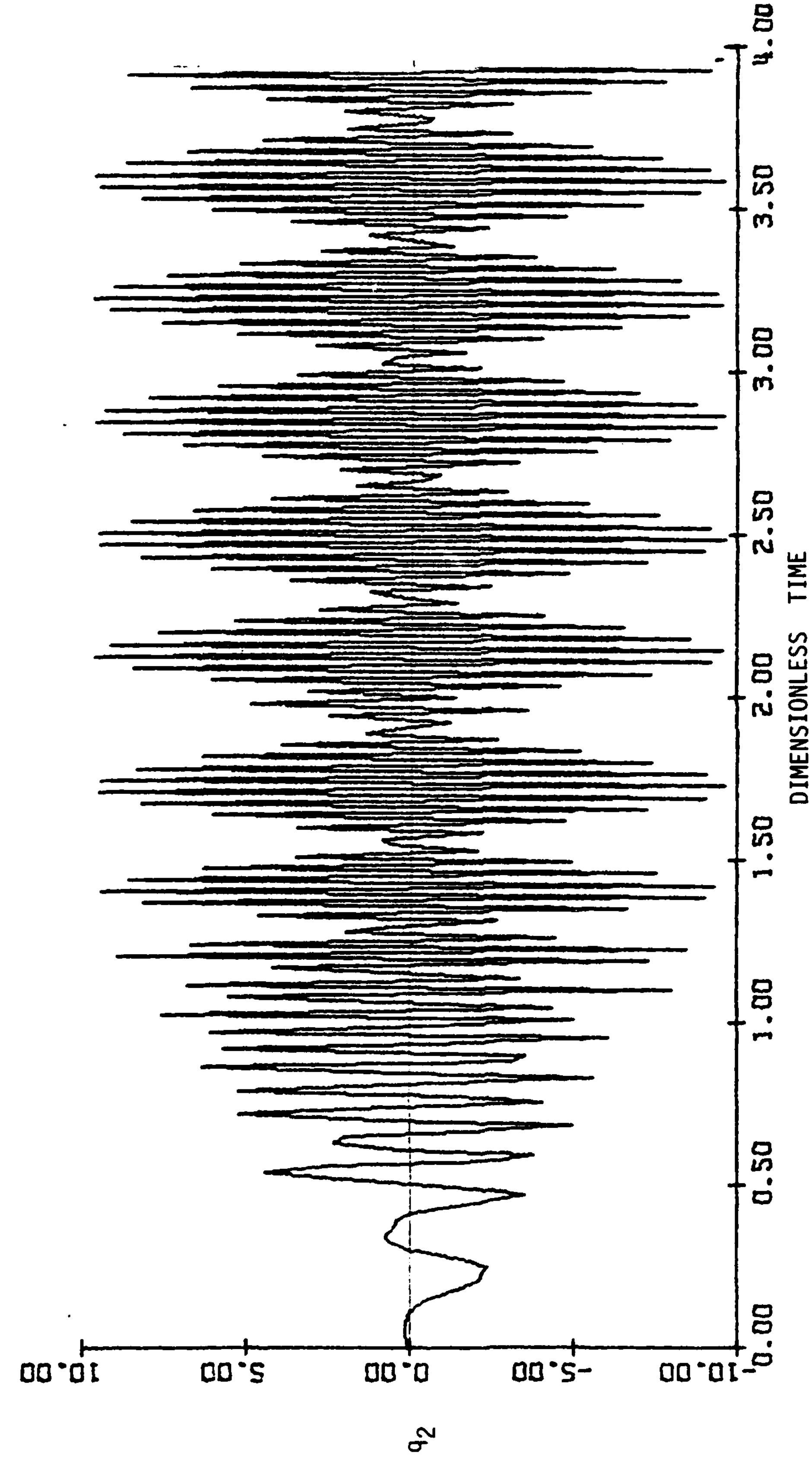


Figure 33b. Second Mode Instability
(μ = 0.5, β =10.0)

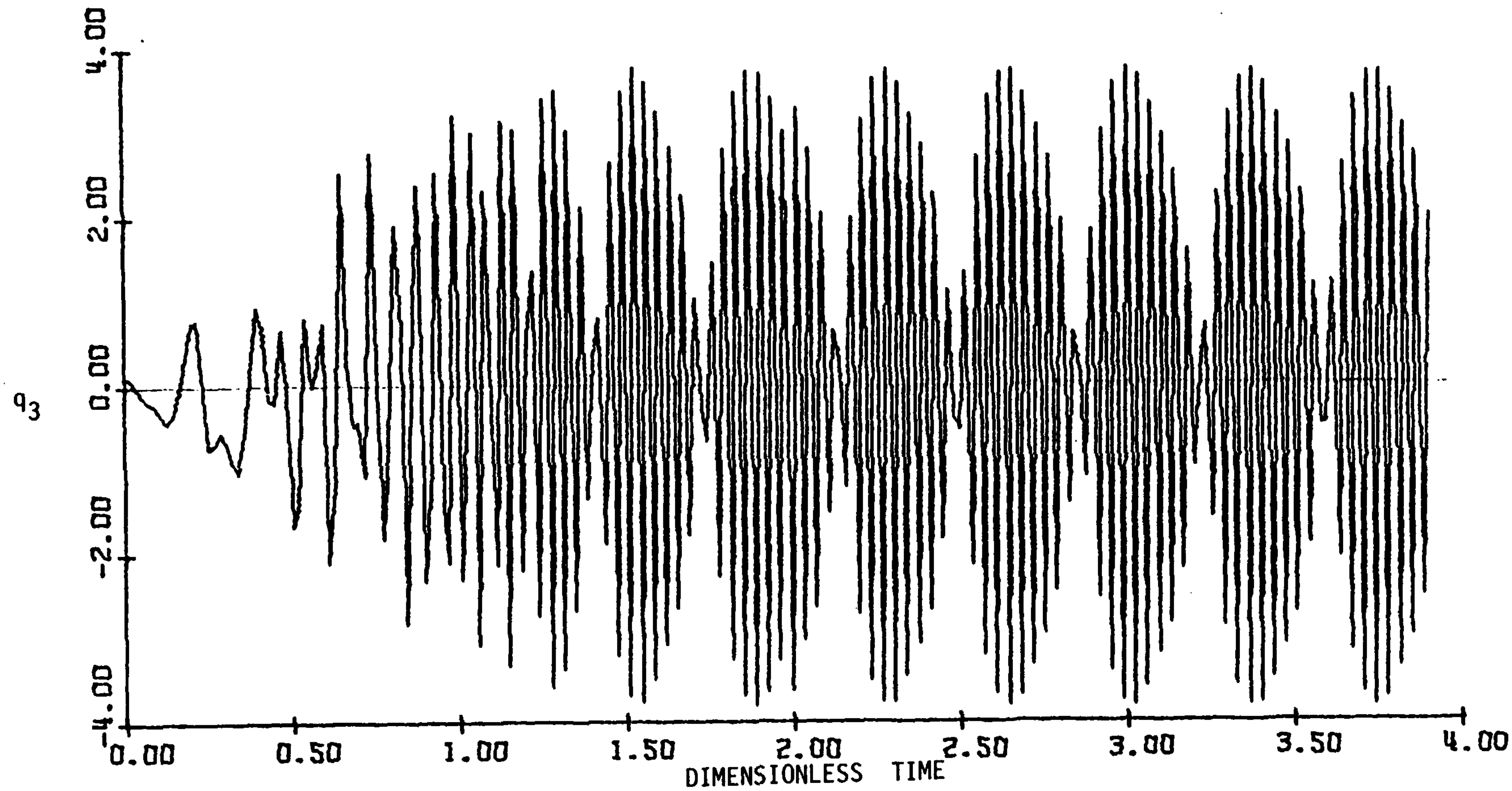


Figure 33c. Third Mode Instability
($\mu = 0.5$, $\beta = 10.0$)

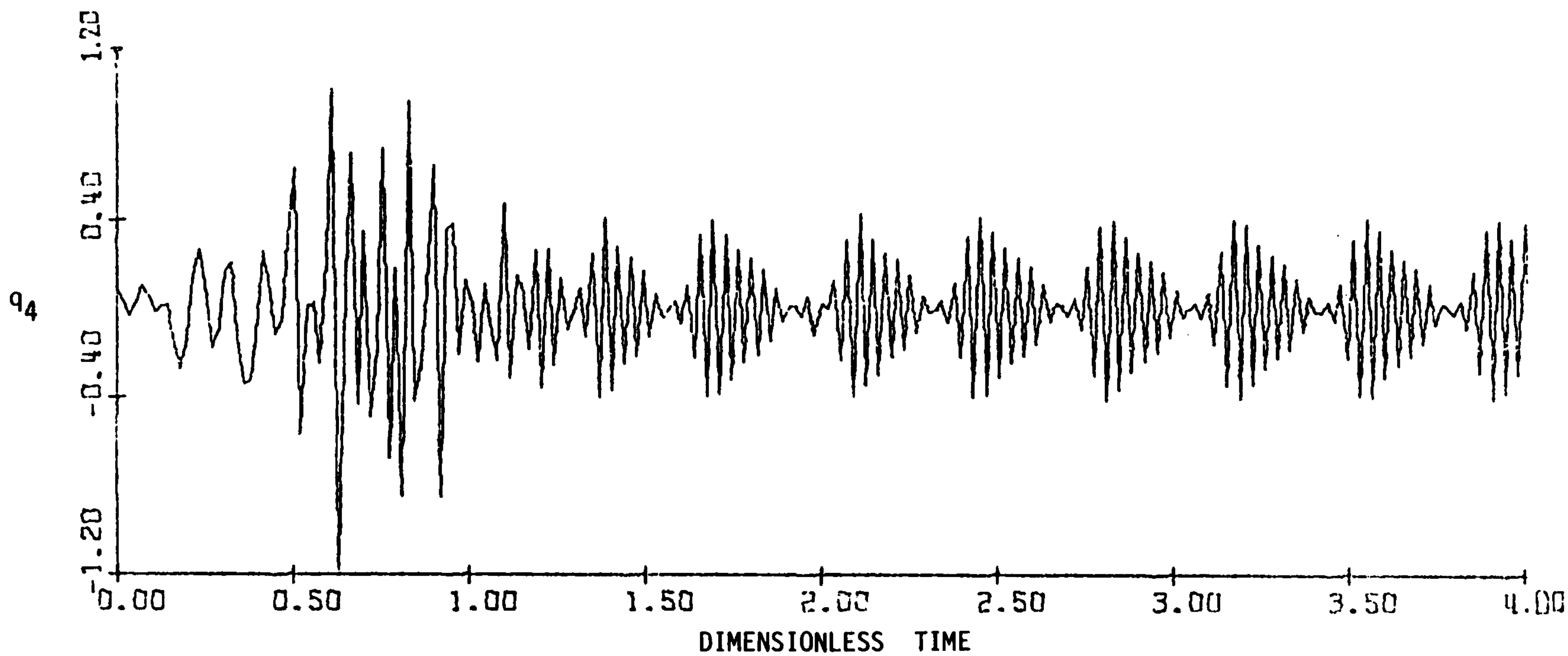


Figure 33d. Fourth Mode Instability
(μ = 0.5, β = 10.0)

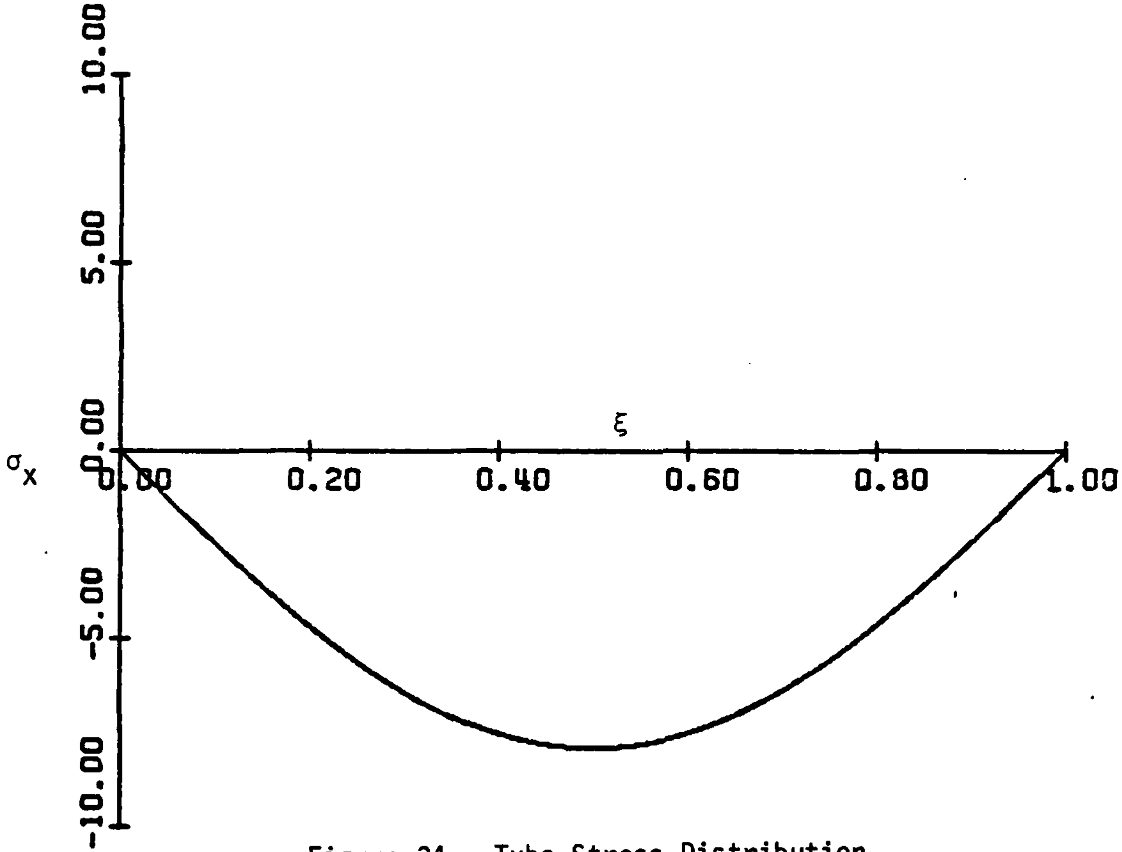


Figure 34. Tube Stress Distribution at Divergence (β = 5.92)

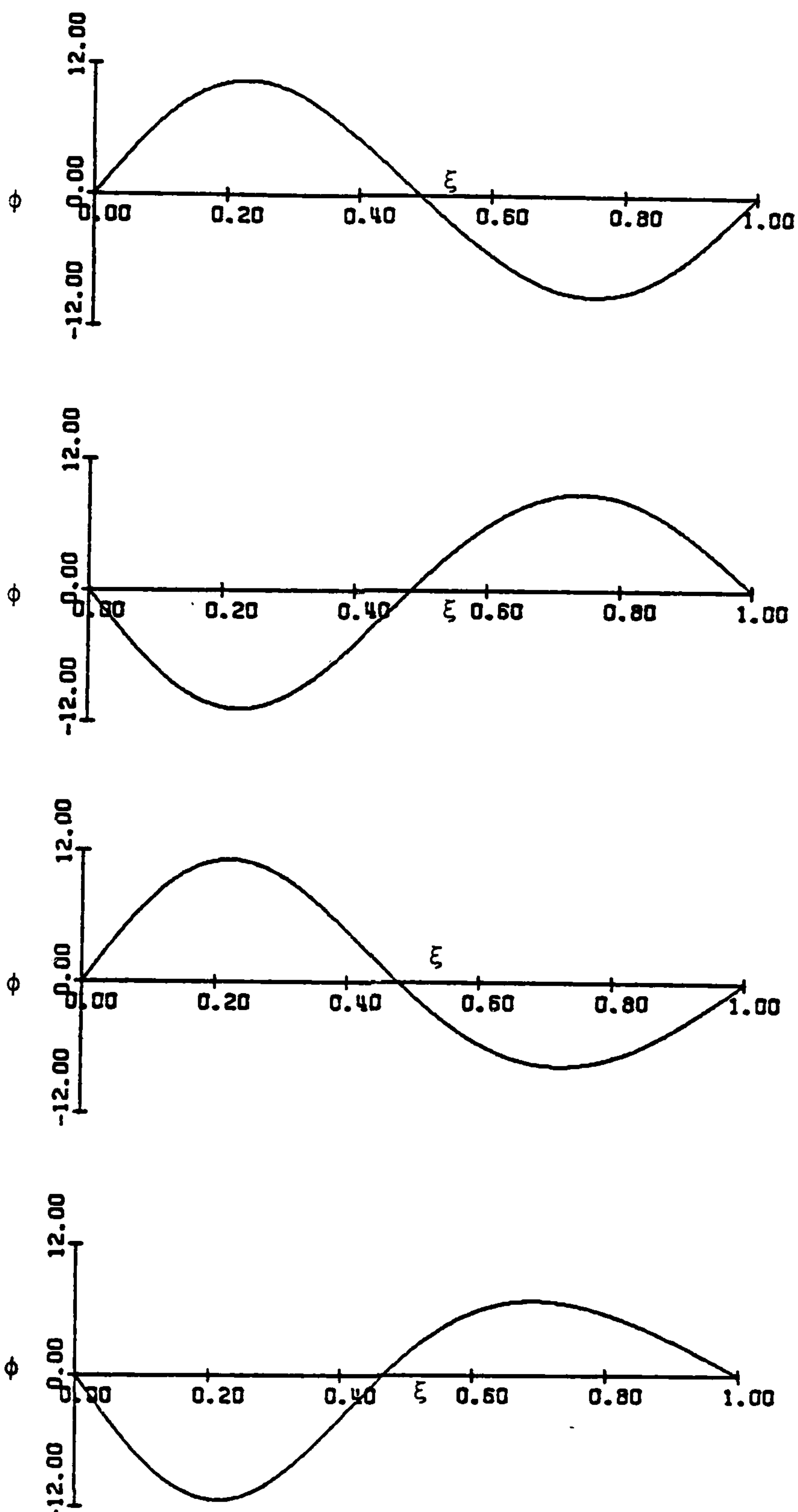


Figure 35a. Tube Deformation Shapes at Various Time Intervals (β = 10.0)

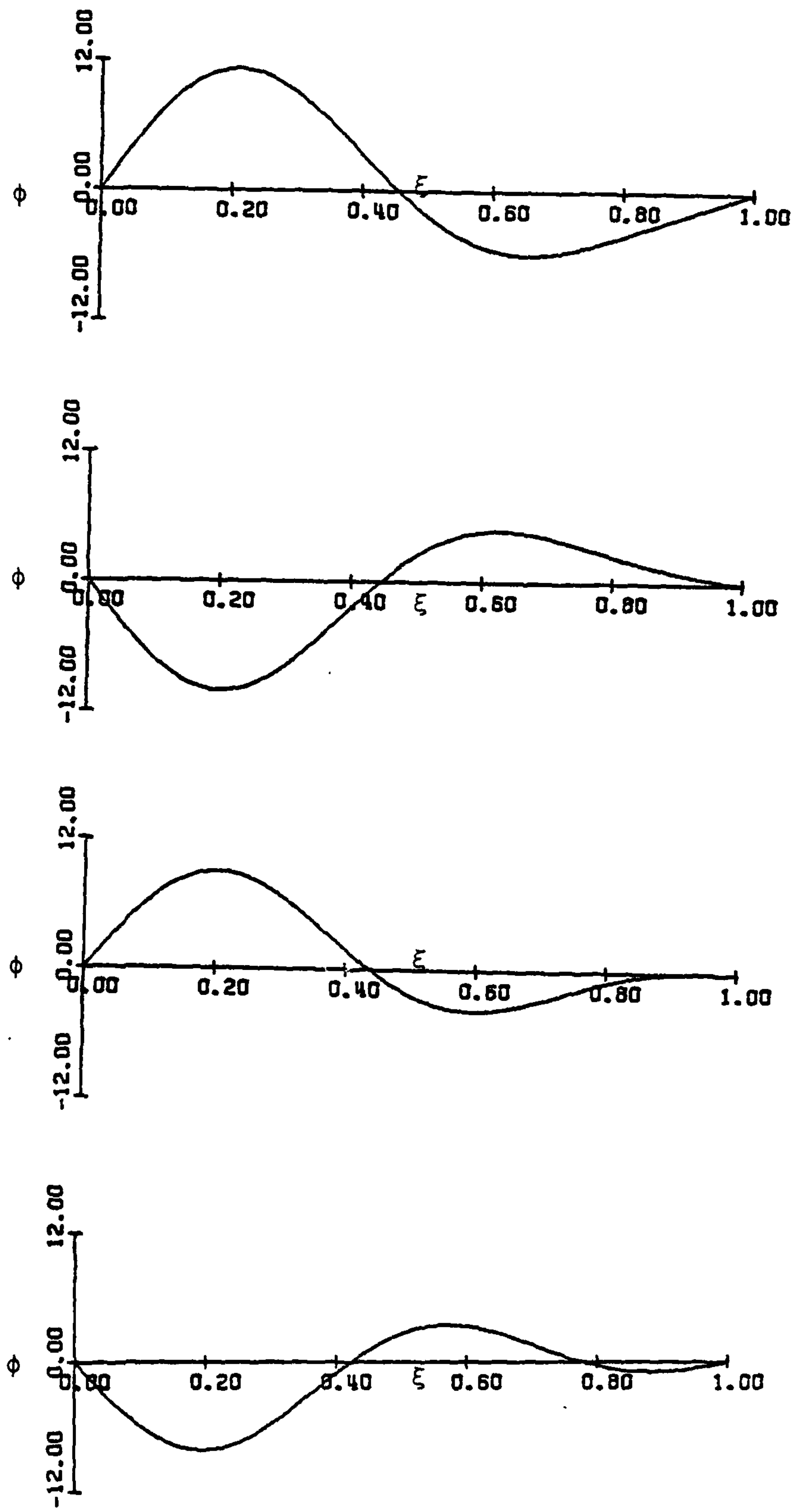


Figure 35b. Tube Deformation Shapes at Various Time Intervals (β = 10.0)

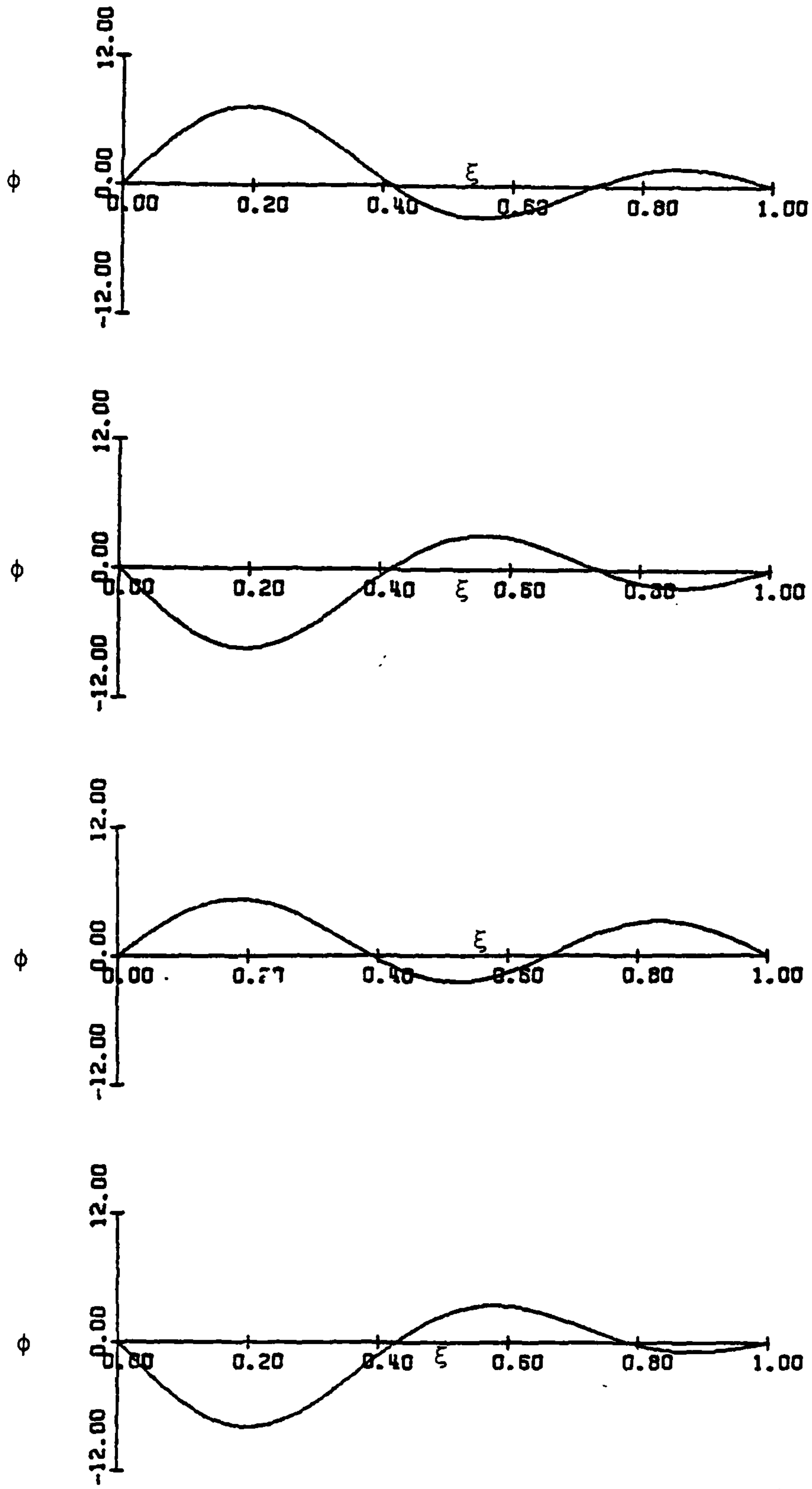


Figure 35c. Tube Deformation Shapes at Various Time Intervals (β = 10.0)

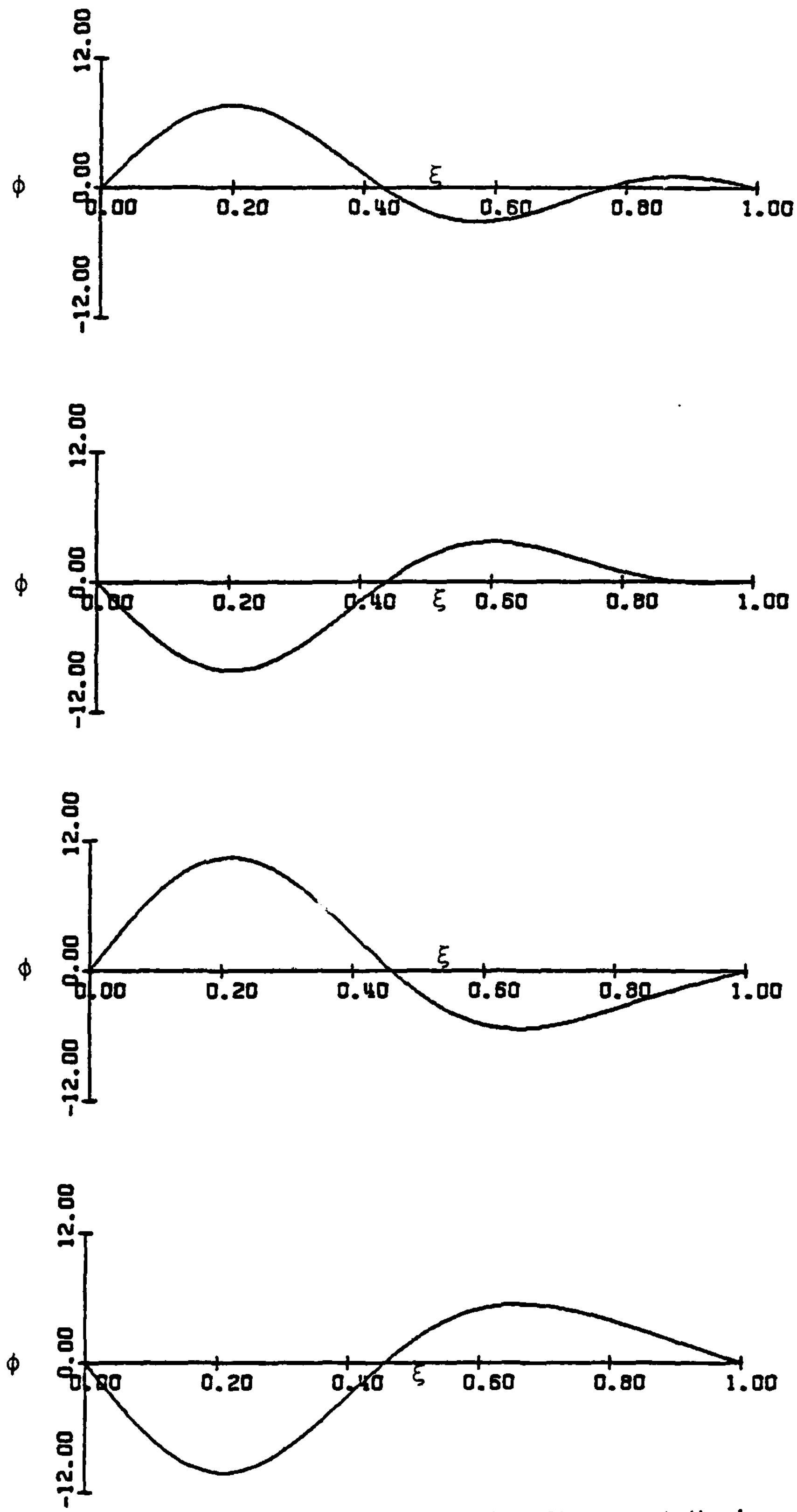


Figure 35d. Tube Deformation Shapes at Various Time Intervals ($\beta = 10.0$)

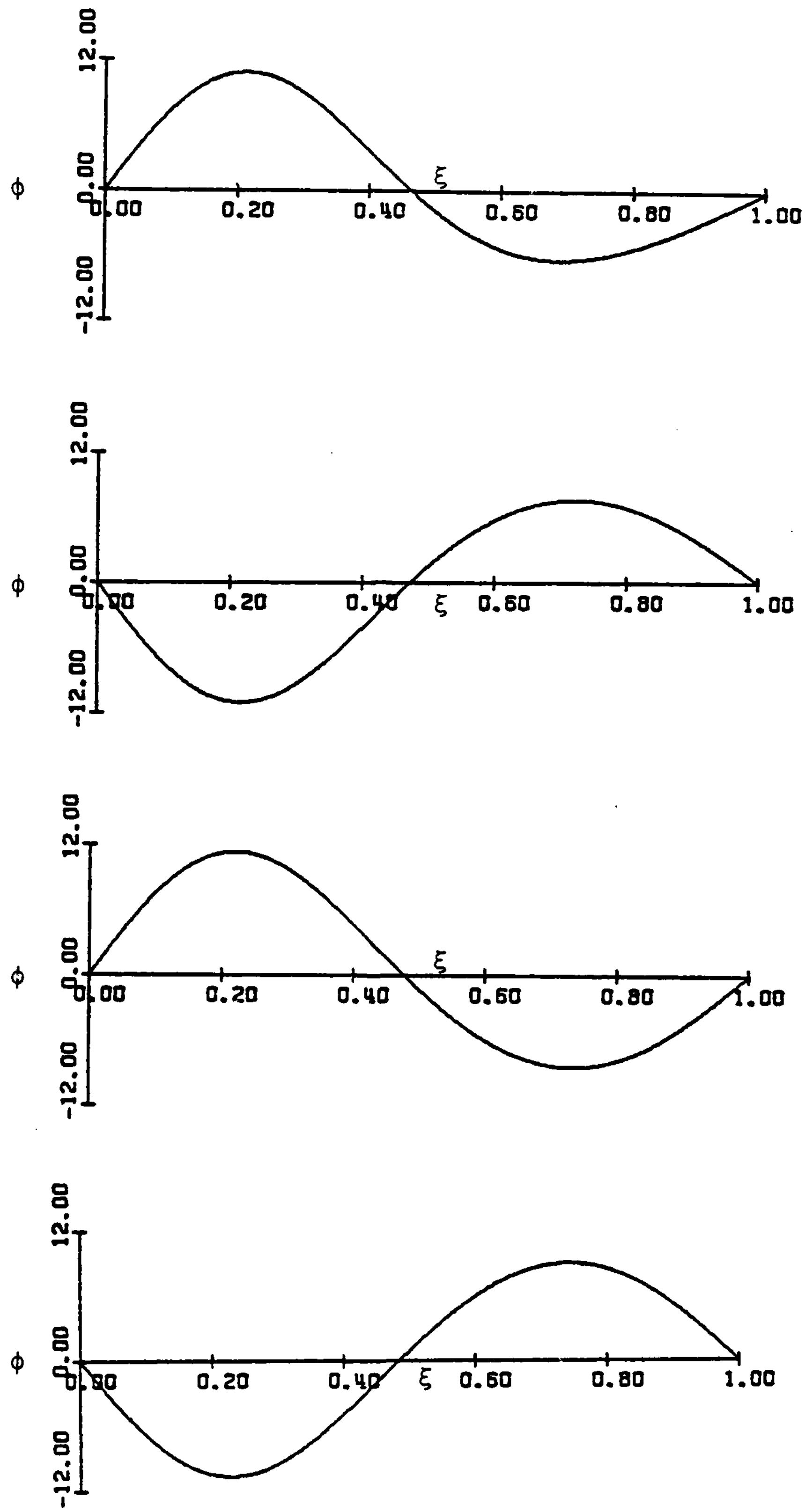


Figure 35e. Tube Deforamtion Shapes at Various Time Intervals (β = 10.0)

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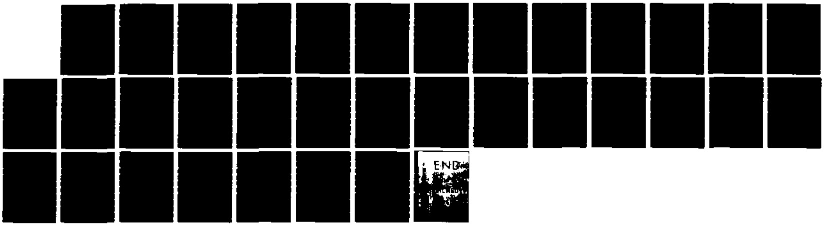

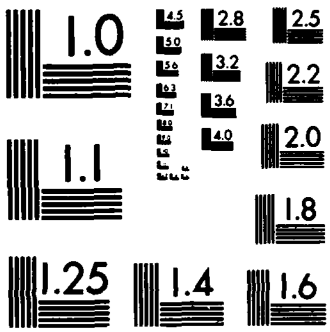

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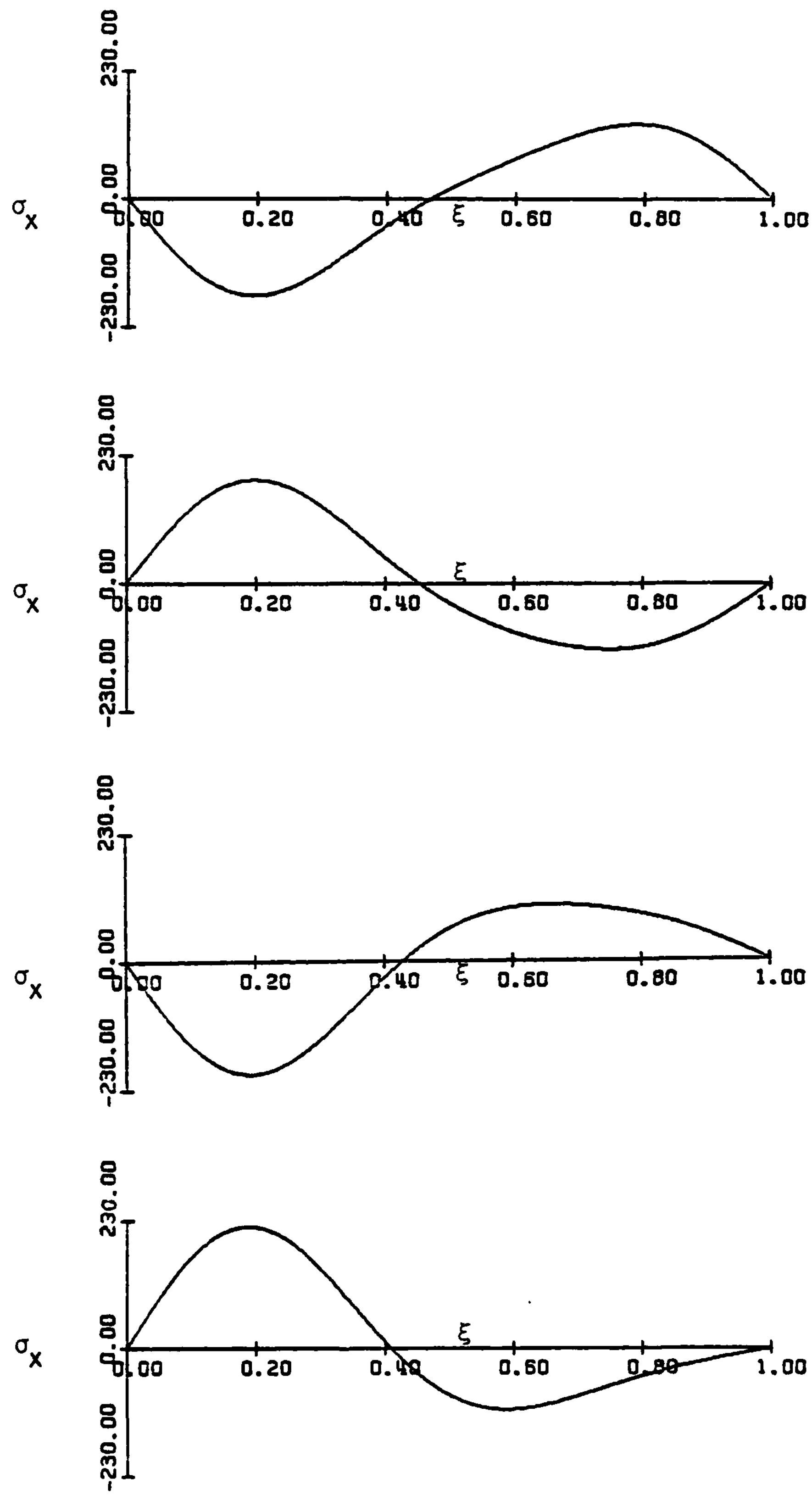

Figure 36a. Tube Stress Distribution at Various Times (β = 10.0)

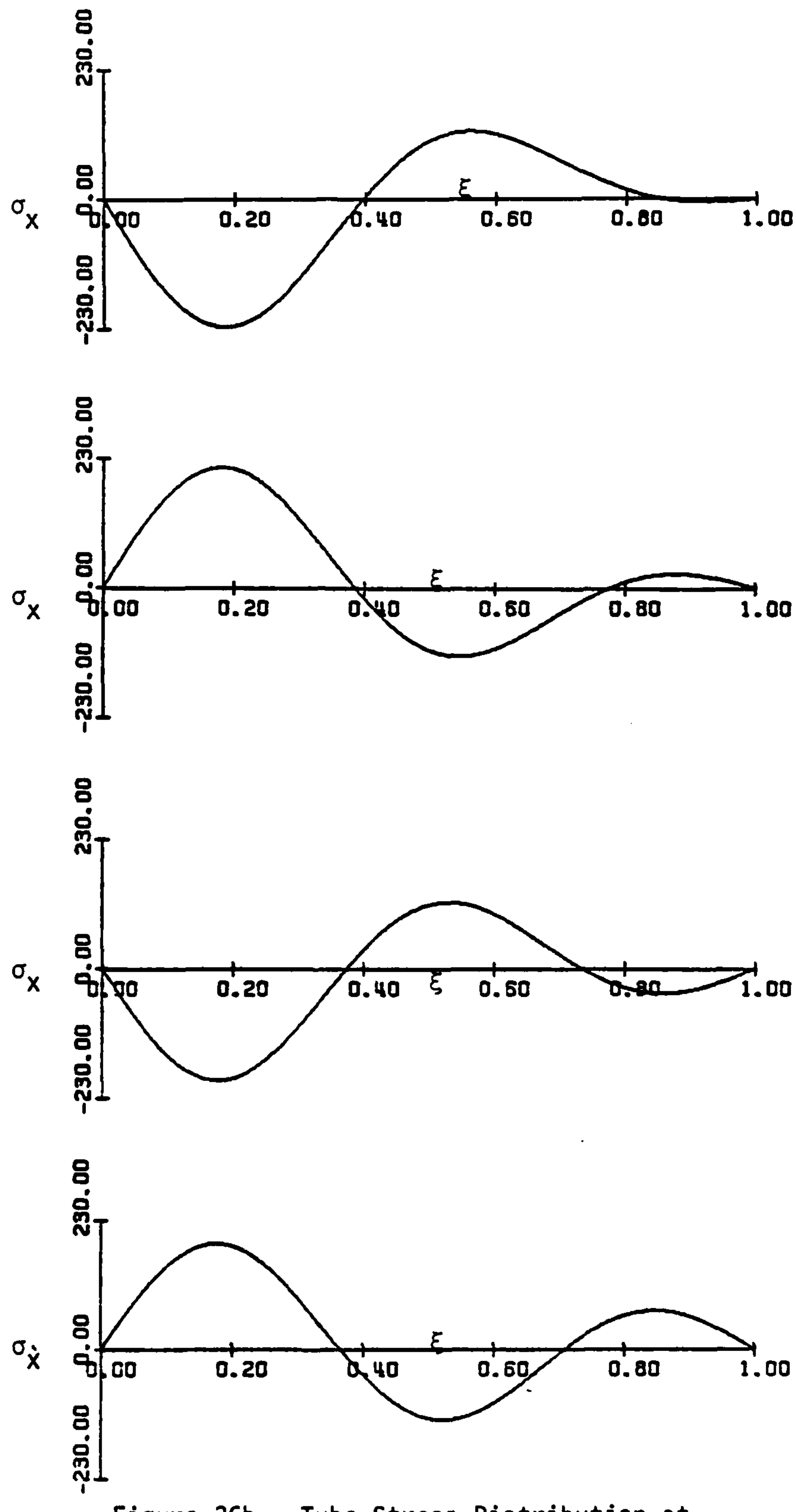

Figure 36b. Tube Stress Distribution at Various Times (β = 10.0)

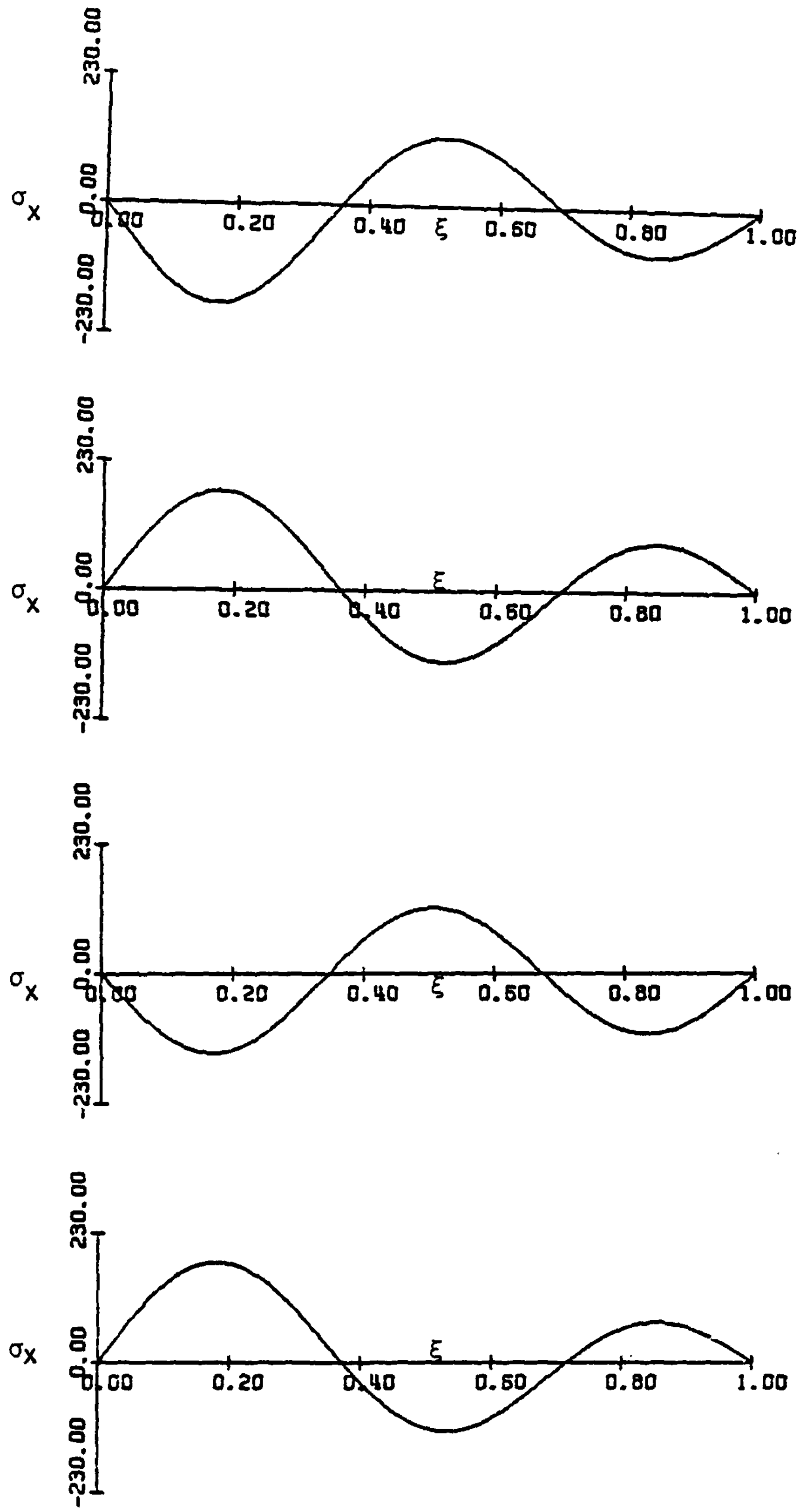

Figure 36c. Tube Stress Distribution at Various Times (β = 10.0)

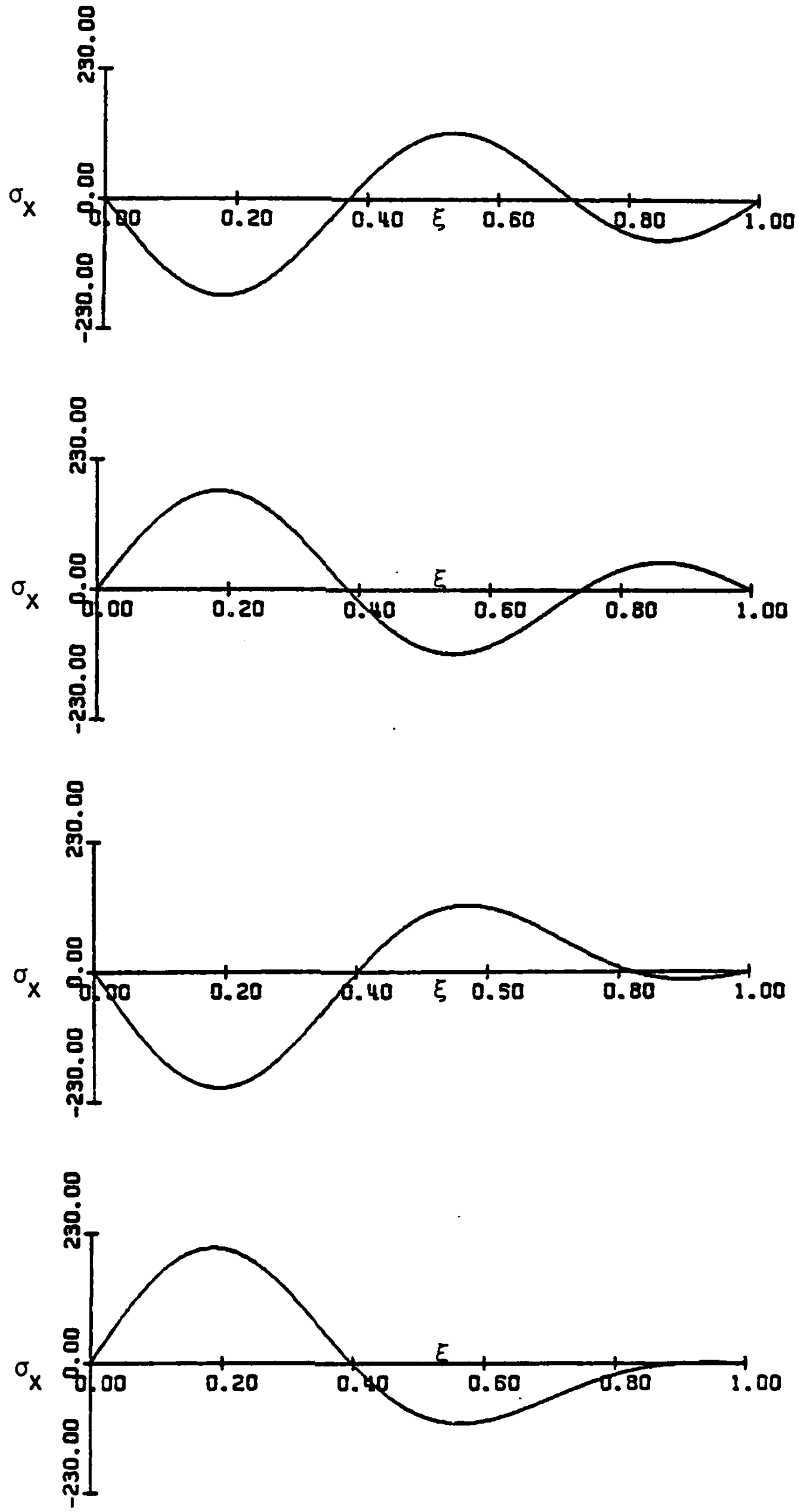

Figure 36d. Tube Stress Distribution at Various Times (β = 10.0)

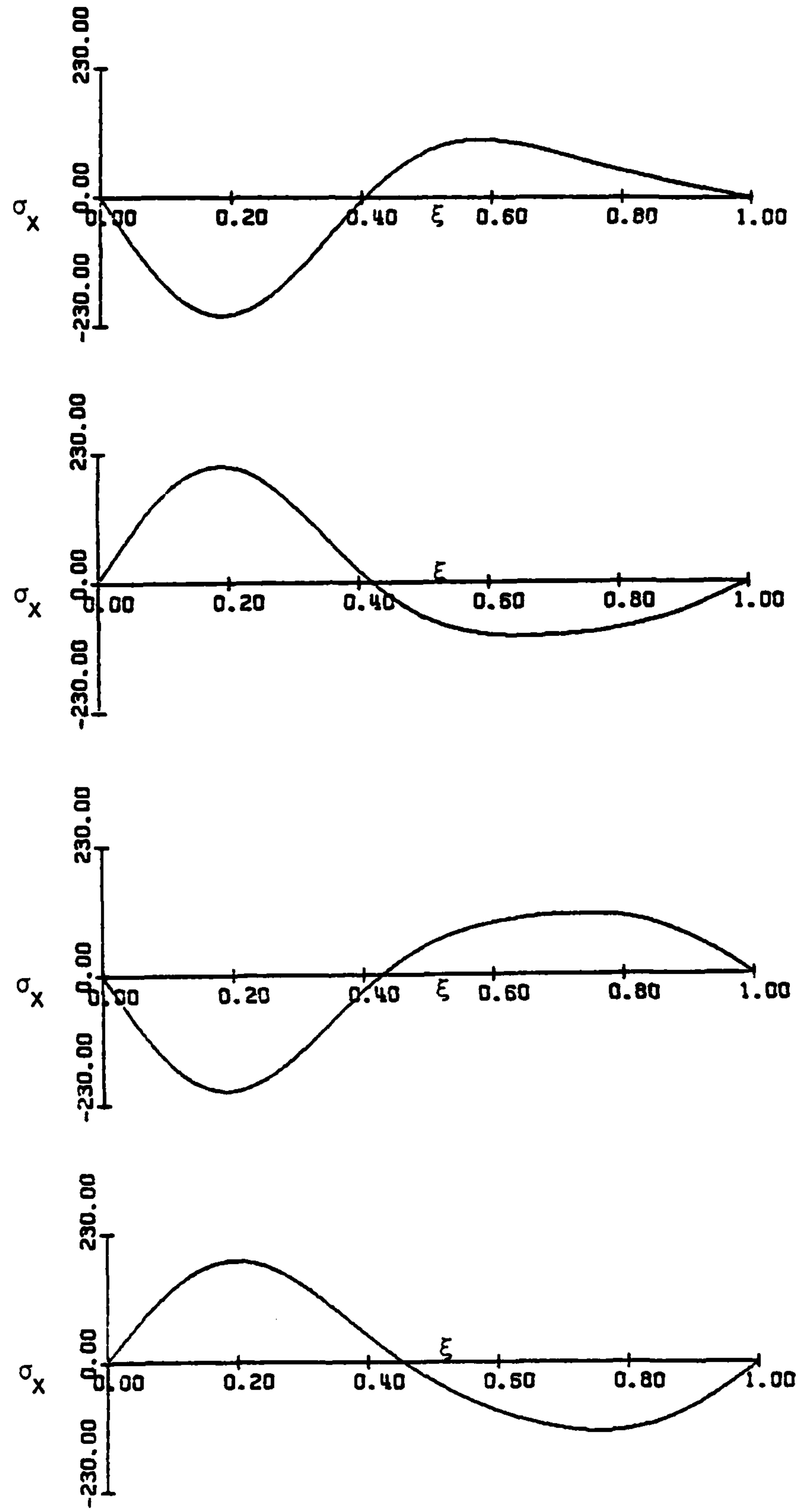

Figure 36e. Tube Stress Distribution at Various Times (β = 10.0)

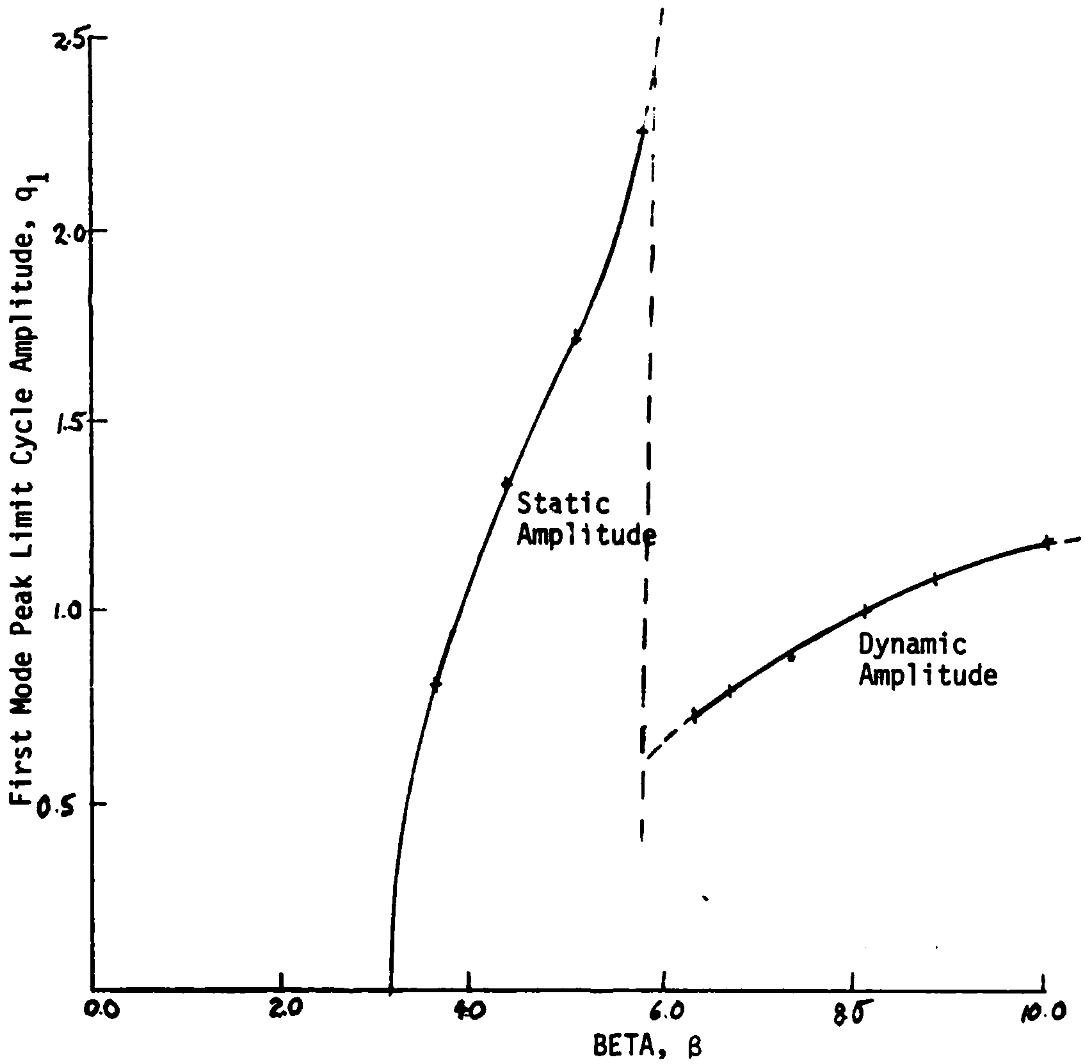

Figure 37a. Effects of β on Peak Amplitude
(μ = 0.5)

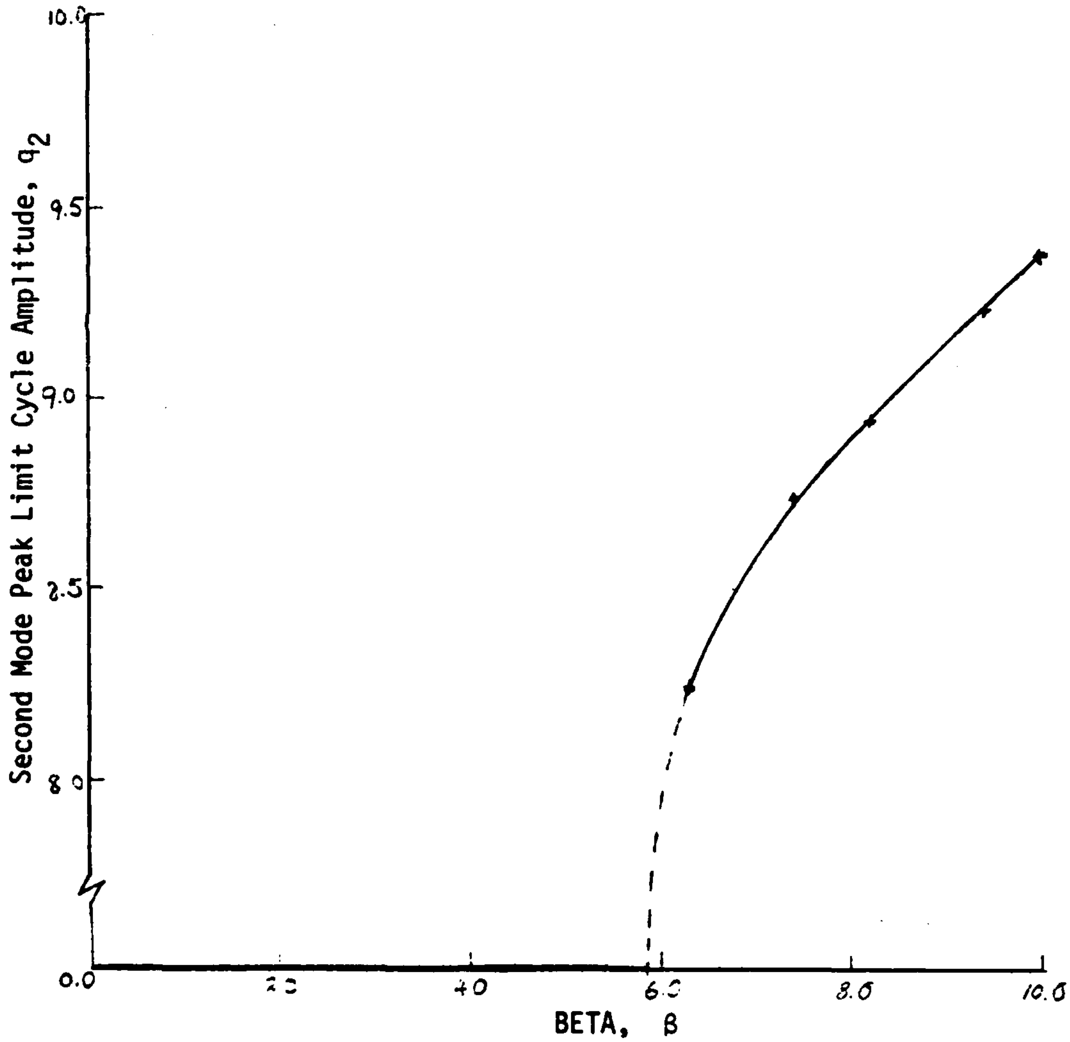

Figure 37b. Effects of β on Peak Amplitude
($\mu = 0.5$)

APPENDIX I

Solving the Energy Equation

Substituting equations (7), (8), and (11) into equation (1) gives

$$\frac{1}{2}\,\delta \int_{t_1}^{t_2}\int_{o}^{\ell} \{m(\frac{\partial y}{\partial t})^2 + m_f[(\frac{\partial y}{\partial t})^2 + 2V\,\frac{\partial y}{\partial t}\,\frac{\partial y}{\partial x} + V^2\,(\frac{\partial y}{\partial x})^2 + V^2]$$

$$- EI\,(\frac{\partial^2 y}{\partial x^2})^2 + mg(\ell - x)\,(\frac{\partial y}{\partial x})^2\}\,dxdt - \int_{t_1}^{t_2}\int_{o}^{\ell} 2\zeta_1\overline{\omega}_1 m\,\frac{\partial y}{\partial t}\,\partial y\,dxdt$$

$$+ \int_{t_1}^{t_2} m_f\,[\frac{\partial y}{\partial t}\,(\ell) + V\,\frac{\partial y}{\partial x}\,(\ell)]\cdot dt\,\delta y\,(\ell) = 0 \qquad \text{(A-1)}$$

In the calculus of variations, δ may be considered as a linear differential operator. Examples:

$$\delta \int_{t_1}^{t_2}\int_{o}^{\ell} m(\frac{\partial y}{\partial t})^2\,dxdt = \int_{t_1}^{t_2}\int_{o}^{\ell} 2m\,\frac{\partial y}{\partial t}\,\delta\,\frac{\partial y}{\partial t}\,dxdt \qquad \text{(A-2)}$$

$$\delta \int_{t_1}^{t_2}\int_{o}^{\ell} m_f V^2\,dxdt = \int_{t_1}^{t_2}\int_{o}^{\ell} \delta\,(m_f V^2)\,dxdt = 0 \qquad \text{(A-3)}$$

$$\delta \int_{t_1}^{t_2}\int_{o}^{\ell} 2m_f V\,\frac{\partial y}{\partial t}\,\frac{\partial y}{\partial x}\,dxdt = \int_{t_1}^{t_2}\int_{o}^{\ell} 2m_f V\,(\frac{\partial y}{\partial t}\,\delta\,\frac{\partial y}{\partial x} + \frac{\partial y}{\partial x}\,\delta\,\frac{\partial y}{\partial t})\,dxdt \qquad \text{(A-4)}$$

Similarly, performing variation on equation (A-1) and then rearranging the terms yields

$$\int_{t_1}^{t_2}\int_{o}^{\ell} \{(m+m_f)\frac{\partial y}{\partial t}\,\delta\,\frac{\partial y}{\partial t}\,dxdt + m_f V\,\frac{\partial y}{\partial t}\,\delta\,\frac{\partial y}{\partial x} + m_f V\,\frac{\partial y}{\partial x}\,\delta\,\frac{\partial y}{\partial t}$$

$$+ m_f V^2\,\frac{\partial y}{\partial x}\,\delta\,\frac{\partial y}{\partial x} - EI\,\frac{\partial^2 y}{\partial x^2}\,\delta\,\frac{\partial^2 y}{\partial x^2} - mg(\ell-x)\,\frac{\partial y}{\partial x}\,\delta\,\frac{\partial y}{\partial x}$$

$$- 2\zeta_1\bar{\omega}_1 m\,\frac{\partial y}{\partial t}\,\delta y\}\,dxdt - \int_{t_1}^{t_2} m_f V\,[\frac{\partial y}{\partial t}(\ell) + V\,\frac{\partial y}{\partial x}(\ell)]\,\delta y(\ell)\,dt = 0 \tag{A-5}$$

Due to the tedious mathematics involved, only representative terms of equation (A-5) will be worked out completely and shown below.

Integration by parts:

$$\int_{t_1}^{t_2}\int_{o}^{\ell}(m+m_f)\,\frac{\partial y}{\partial t}\,\delta\,\frac{\partial y}{\partial t}\,dxdt = (m+m_f)\,\{\int_{o}^{\ell}\,[\frac{\partial y}{\partial t}\,\delta y]\int_{t_1}^{t_2}\,dx$$

$$- \int_{o}^{\ell}\int_{t_1}^{t_2}\frac{\partial^2 y}{\partial t^2}\,\delta y\,dxdt\} \tag{A-6}$$

$$\int_{t_1}^{t_2}\int_{o}^{\ell}(m_f V\,\frac{\partial y}{\partial t}\,\delta\,\frac{\partial y}{\partial x})\,dxdt = m_f V\,\{\int_{t_1}^{t_2}\,[\frac{\partial y}{\partial t}\,\delta y]_{o}^{\ell}\,dt -$$

$$\int_{t_1}^{t_2}\int_{o}^{\ell}\frac{\partial^2 y}{\partial x\partial t}\,\delta y\,dxdt\} \tag{A-7}$$

$$\int_{t_1}^{t_2}\int_{o}^{\ell}EI\,\frac{\partial^2 y}{\partial x^2}\,\delta\,\frac{\partial^2 y}{\partial x^2}\,dxdt = EI\,\{\int_{t_1}^{t_2}\,[\frac{\partial^2 y}{\partial x^2}\,\delta\,\frac{\partial y}{\partial x}]_{o}^{\ell}\,dt - \int_{t_1}^{t_2}\int_{o}^{\ell}\frac{\partial^3 y}{\partial x^3}\,\delta\,\frac{\partial y}{\partial x}\,dx\,dt\}$$

$$= EI \left\{ \int_{t_1}^{t_2} \left[\frac{\partial^2 y}{\partial x^2} \delta \frac{\partial y}{\partial x}\right]_o^{\ell} dt - \int_{t_1}^{t_2} \left[\frac{\partial^3 y}{\partial x^3} \delta y\right]_o^{\ell} dt + \int_{t_1}^{t_2} \int_o^{\ell} \frac{\partial^4 y}{\partial x^4} \delta y \, dx \, dt \right\} \quad \text{(A-8)}$$

$$\int_{t_1}^{t_2} \int_o^{\ell} m_f V \frac{\partial y}{\partial x} \delta \frac{\partial y}{\partial x} dxdt = \int_o^{\ell} \left\{ \left[-m_f V \frac{\partial y}{\partial x} \delta y\right]_{t_1}^{t_2} - \int_{t_1}^{t_2} m_f V \frac{\partial^2 y}{\partial x \partial t} \delta y \, dt \right\} dx \quad \text{(A-9)}$$

$$\int_o^{\ell} m_f V^2 \frac{\partial y}{\partial x} \delta \frac{\partial y}{\partial x} dx = m_f V^2 \frac{\partial y}{\partial x} \delta y \Big|_{x=0}^{\ell} - \int_o^{\ell} m_f V^2 \frac{\partial^2 y}{\partial x^2} \delta y \, dx \quad \text{(A-10)}$$

$$\int_o^{\ell} - mg(\ell-x) \frac{\partial y}{\partial x} \delta \frac{\partial y}{\partial x} dx = -mg(\ell-x) \frac{\partial y}{\partial x} \delta y \Big|_{x=0}^{\ell} + \int_o^{\ell} mg \frac{\partial}{\partial x} \left[(\ell-x) \frac{\partial y}{\partial x}\right] \delta y \, dx \quad \text{(A-11)}$$

and $[\delta y]_{t_1}^{t_2} = 0$ (See Ref. 6)

To satisfy Eq. (A-5), the integrands have to be zero. The sum of all the integrands of the double integrals give the partial differential equation for the problem,

$$EI \frac{\partial^4 y}{\partial x^4} + (m+m_f) \frac{\partial^2 y}{\partial t^2} + 2m_f V \frac{\partial^2 y}{\partial t \partial x} + m_f V^2 \frac{\partial^2 y}{\partial x^2} + 2m\zeta_1 \bar{\omega}_1 \frac{\partial y}{\partial t} - mg \frac{\partial}{\partial x} \left[(\ell-x) \frac{\partial y}{\partial x}\right] = 0 \quad \text{(A-12)}$$

Also the several end point terms must be zero. Thus

$$EI \frac{\partial^2 y}{\partial x^2} \delta\left(\frac{\partial y}{\partial x}\right)\Big|_{x=0,\ \ell} = 0 \quad \text{(A-13)}$$

For a cantilevered tube (A-13) is satisfied by

$$\left.\frac{\partial y}{\partial x}\right|_{x=0} = 0$$

(A-13c)

and

$$EI \left.\frac{\partial^2 y}{\partial x^2}\right|_{x=\ell} = 0$$

For a simply-supported tube,

$$EI \left.\frac{\partial^2 y}{\partial x^2}\right|_{x=0,\ \ell} = 0 \qquad \text{(A-13ss)}$$

The other end point conditions are

$$\left.\left[EI \frac{\partial^3 y}{\partial x^3} - mg(\ell-x) \frac{\partial y}{\partial x}\right] \delta y\right|_{x=0,\ \ell} = 0 \qquad \text{(A-14)}$$

For a cantilevered tube, (A-14) is satisfied by

$$y|_{x=0} = 0$$

(A-14c)

and

$$\left[EI \frac{\partial^3 y}{\partial x^3} - mg(\ell-x) \frac{\partial y}{\partial x}\right]_{x=0} = 0$$

For a simply-supported tube,

$$y|_{x=0,\ \ell} = 0 \qquad \text{(A-14ss)}$$

It should be noted that in obtaining (A-14) from (A-5) to (A-11), there has been a cancellation of terms involving the last term in (A-5) and the first terms on the right hand side of (A-7) and (A-10) for $x=\ell$ to give the same form in (A-14) for the boundary conditions at <u>both</u> $x=0$ <u>and</u> $x=\ell$.

APPENDIX II

Mode Shape Integrals of a Cantilevered Beam

Integrals containing the characteristic functions and their derivatives are referred to as mode shape integrals in this paper. They can be evaluated by the method of partial integration along with their orthogonality properties. Special formulas for those mode shape integrals which occur frequently in engineering application may be found in references 10 and 11. They are given in terms of α_n and β_n for five common boundary conditions: clamped-clamped, clamped-free, clamped-sup: ɔrted, free-free, and free-supported.

As an illustration, consider

$$c_{hi} = \int_0^1 \gamma_h' (\xi)\, \gamma_i' (\xi)\, d\xi \qquad \text{(A-15)}$$

for a cantilevered uniform beam. Combining formulas (6) and (16) of reference (7) yields.

$$c_{hi} = \begin{cases} \alpha_h\beta_h\,(2 + \alpha_h\beta_h), & h = i \\ \\ \dfrac{4\beta_h\beta_i}{\beta_h^4 - \beta_i^4}\,[(-1)^{h+i}\,(\alpha_i\beta_h^3 - \alpha_h\beta_i^3) - \beta_h\beta_i\,(\alpha_h\beta_h - \alpha_i\beta_i), & h \neq i \end{cases}$$

The corresponding α's and β's can be found in reference 11. They are

n	α_n	β_n
1	0.7341	1.8751
2	1.0185	4.6940
3	0.9992	7.8546
4	1.0000	10.9955

Numerical values of c_{hi} are given in Table 2. The computations were done on the IBM 360/91

APPENDIX III

<u>Radius of gyration, r</u>

(i) For a thin cylindrical tube, the area moment is

$$I = \int z^2 \, da \equiv r^2 a$$

Now $$a = 2\pi \bar{R} t$$

and $$da = \bar{R} \cdot t \cdot d\theta$$

Also $$z = \bar{R} \cos \theta$$

Thus $$I = \int_0^{2\pi} \bar{R}^2 \cos^2 \theta \, \bar{R} \, d\theta \, t$$

$$= \bar{R}^3 t \int_0^{\pi} \cos^2 \theta \, d\theta$$

$$= \bar{R}^3 t \pi$$

Hence $$r^2 a = \bar{R}^3 t \pi$$

Solving, $$r^2 = \frac{\bar{R}^3 t \pi}{2\pi \bar{R} t}$$

or $$r^2 = \frac{\bar{R}^2}{2}$$

$$r = \sqrt{\frac{\bar{R}^2}{2}}$$

(II) For a tube with a thick wall,

$$I = \int_0^{2\pi} \int_{\bar{R}_1}^{\bar{R}_2} \bar{R}^2 \cos^2 \theta \, \bar{R} \, d\bar{R} \, d\theta$$

$$= \int_0^{2\pi} [(\overline{R}_2^4 - \overline{R}_1^4)/4] \cos^2 \theta \, d\theta$$

$$= \frac{\pi}{4} [\overline{R}_2^4 - \overline{R}_1^4]$$

$$a = \int_0^{2\pi} \int_{\overline{R}_1}^{\overline{R}_2} \overline{R} \, d\overline{R} \, d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} (\overline{R}_2^2 - \overline{R}_1^2) \, d\theta$$

$$= \pi[\overline{R}_2^2 - \overline{R}_1^2]$$

Hence $$r^2 = I/a$$

$$= \frac{\pi[\overline{R}_2^4 - \overline{R}_1^4]}{4\pi[\overline{R}_2^2 - \overline{R}_1^2]}$$

$$= \frac{1}{4} [\overline{R}_2^2 + \overline{R}_1^2]$$

APPENDIX IV

A LISTING OF WATFIV PROGRAMS FOR CANTILEVERED TUBE

(Linear and Nonlinear)

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C
C         ++++++++++++++++++++++++++++++++++++++++++
C         +                                        +
C         +    NUMERICAL SOLUTION FOR STATIC AND   +
C         +   DYNAMIC INSTABILITIES OF A PROPELLANT +
C         +       LINE.  (FOUR MODES ANALYSIS)     +
C         +      NB:  WITH FORMULA OF OPEN TYPES   +
C         +                                        +
C         ++++++++++++++++++++++++++++++++++++++++++
C
          INTEGER II,JJ,N1,N2,I1,I2,I3,I4
          REAL DQ(8,1500)
          REAL DUMMY,MATH
          REAL RMU
          REAL  BETA,MU,NU,M,MF,I,E
          REAL Q1,Q2,Q3,Q4,L1,L2,VF
          REAL Q5,Q6,Q7,Q8
          REAL L3,L4
          REAL L,Z1,T,TMAX,DELT,A11,A12
          REAL A21,A22,B12,B22,B21,B11
          REAL A13,A14,A23,A24
          REAL A31,A32,A33,A34
          REAL A41,A42,A43,A44
          REAL B13,B14,B23,B24
          REAL B31,B32,B33,B34
          REAL B41,B42,B43,B44
          REAL  K(8,8),AQ(8,1500),AT(1500),V(8)
          REAL       TQ1(500),TQ2(500),TT(500)
          REAL TQ3(500),TQ4(500)
C
C              Q1, Q2, Q3, ...  ARE INITIAL CONDITIONS
C              DQ(N)'S ARE PSEUDO INITIAL CONDITIONS
C              L1, L2 ARE MODE SHAPE PARAMETERS
C              L = LENGTH OF PIPE
C              Z1 = CRITICAL DAMPING RATIO
C              M = MASS OF PIPE
C              MF = MASS OF FLUID
C              VF = FLUID VELOCITY
C              I = INERTIA
C              E = MODULUS OF ELASTICITY
C            A11, A12, ..., B11, B12, ...ARE MODE SHAPE INTEGRALS
C
          READ (5,1000) Q1,Q2,Q3,Q4,Q5,Q6
          READ (5,1000) Q7,Q8,L1,L2,L3,L4
     1000 FORMAT (6F10.4)
          READ (5,1001) A11,A12,A21,A22,B11,B12,B21,B22
          READ (5,1001) A13,A14,A23,A24,B13,B14,B23,B24
          READ (5,1001) A31,A32,A33,A34,A41,A42,A43,A44
          READ (5,1001) B31,B32,B33,B34,B41,B42,B43,B44
     1001 FORMAT (8F10.4)
     998  READ (5,1111,END=999) VF,L,Z1,T,DELT,TMAX
     1111 FORMAT (F12.4,5F10.4)
          READ (5,1002) M,MF,I,E
     1002 FORMAT (4E10.3)
C
C              BETA = DIMENSIONLESS FLUID VELOCITY
C              MU = MASS RATIO
```

```
C            NU = VISCOUS DAMPING COEFFICIENT
C
          BETA = VF*SQRT(MF*L**2./(E*I))
          MU = MF/(MF+M)
          NU = 7.04*Z1*SQRT(1.-MU)
C
          WRITE (6,1012)
          WRITE (6,1003) A11,A12,A13,A14,A21,A22,A23,A24
     1003 FORMAT (' ','A(1,J)=',4F20.4,/,' ','A(2,J)=',4F20.4)
          WRITE (6,3011) A31,A32,A33,A34,A41,A42,A43,A44
     3011 FORMAT (' ','A(3,J)=',4F20.4,/,' ','A(4,J)=',4F20.4)
          WRITE (6,3001) B11,B12,B13,B14,B21,B22,B23,B24
     3001 FORMAT (' ','B(1,J)=',4F20.4,/,' ','B(2,J)=',4F20.4)
          WRITE (6,3111) B31,B32,B33,B34,B41,B42,B43,B44
     3111 FORMAT (' ','B(3,J)=',4F20.4,/,' ','B(4,J)=',4F20.4)
          WRITE (6,1004) Q1,Q2,Q3,Q4,Q5,Q6,Q7,Q8
     1004 FORMAT (' ','Q(N)=',4F20.4,/,6X,4F20.4)
          WRITE (6,1005) BETA,MU,NU
     1005 FORMAT (' ','BETA=',1F15.4,5X,'MU=',1F15.4,5X,'NU=',1F15.4)
          WRITE (6,1006) VF,L1,L2,L3,L4,L,Z1
     1006 FORMAT (' ','VF=',1F15.4,5X,'L1 ... ARE',4F10.5,/,' ','L=',1F10.
         15X,'Z1=',1F10.4)
          WRITE (6,1007)  M,MF,I,E
     1007 FORMAT (' ','M,MF,I,E ARE',4E20.3)
          WRITE (6,1008)
     1008 FORMAT (//,30X,'MATRIX IS',/)
C
          RMU = SQRT(MU)
C
C         ++++++++++++++++++++++++++++++++++++++++++++
C         +                                          +
C         +            ENTRIES OF MATRIX             +
C         +                                          +
C         ++++++++++++++++++++++++++++++++++++++++++++
C
          DQ(1,1) = 0.0
          DQ(1,2) = 0.0
          DQ(2,1) = 0.0
          DQ(2,2) = 0.0
          DQ(3,1) = 0.0
          DQ(3,2) = 0.0
          DQ(4,1) = 0.0
          DQ(4,2) = 0.0
          DQ(5,1) = 0.0
          DQ(5,2) = 0.0
          DQ(6,1) = 0.0
          DQ(6,2) = 0.0
          DQ(7,1) = 0.0
          DQ(7,2) = 0.0
          DQ(8,1) = 0.0
          DQ(8,2) = 0.0
          K(1,1) = 0.0
          K(1,2) = 0.0
          K(1,3) = 0.0
          K(1,4) = 0.0
          K(1,5) = 1.0
          K(1,6) = 0.0
          K(1,7) = 0.0
          K(1,8) = 0.0
          K(2,1) = 0.0
```

```
      K(2,2) = 0.0
      K(2,3) = 0.0
      K(2,4) = 0.0
      K(2,5) = 0.0
      K(2,6) = 1.0
      K(2,7) = 0.0
      K(2,8) = 0.0
      K(3,1) = 0.0
      K(3,2) = 0.0
      K(3,3) = 0.0
      K(3,4) = 0.0
      K(3,5) = 0.0
      K(3,6) = 0.0
      K(3,7) = 1.0
      K(3,8) = 0.0
      K(4,1) = 0.0
      K(4,2) = 0.0
      K(4,3) = 0.0
      K(4,4) = 0.0
      K(4,5) = 0.0
      K(4,6) = 0.0
      K(4,7) = 0.0
      K(4,8) = 1.0
      K(5,1) = -(L1**4.+BETA**2.*B11)
      K(5,2) = -BETA**2.*B12
      K(5,3) = -BETA**2.*B13
      K(5,4) = -BETA**2.*B14
      K(5,5) = -(2.*BETA*RMU*A11+NU)
      K(5,6) = -2.*BETA*RMU*A12
      K(5,7) = -2.*BETA*RMU*A13
      K(5,8) = -2.*BETA*RMU*A14
      K(6,1) = -BETA**2.*B21
      K(6,2) = -(L2**4.+BETA**2.*B22)
      K(6,3) = -BETA**2.*B23
      K(6,4) = -BETA**2.*B24
      K(6,5) = -2.*BETA*RMU*A21
      K(6,6) = -(2.*BETA*RMU*A22+NU)
      K(6,7) = -2.*BETA*RMU*A23
      K(6,8) = -2.*BETA*RMU*A24
      K(7,1) = -BETA**2.*B31
      K(7,2) = -BETA**2.*B32
      K(7,3) = -(L3**4.+BETA**2.*B33)
      K(7,4) = -BETA**2.*B34
      K(7,5) = -2.*BETA*RMU*A31
      K(7,6) = -2.*BETA*RMU*A32
      K(7,7) = -(2.*BETA*RMU*A33+NU)
      K(7,8) = -2.*BETA*RMU*A34
      K(8,1) = -BETA**2.*B41
      K(8,2) = -BETA**2.*B42
      K(8,3) = -BETA**2.*B43
      K(8,4) = -(L4**4.+BETA**2.*B44)
      K(8,5) = -2.*BETA*RMU*A41
      K(8,6) = -2.*BETA*RMU*A42
      K(8,7) = -2.*BETA*RMU*A43
      K(8,8) = -(2.*BETA*RMU*A44+NU)
C
      WRITE (6,1009) ((K(II,JJ), JJ=1,8), II=1,8)
 1009 FORMAT (' ',8F12.4)
C
      N1 = 1
```

```
      AQ(1,N1) = Q1
      AQ(2,N1) = Q2
      AQ(3,N1) = Q3
      AQ(4,N1) = Q4
      AQ(5,N1) = Q5
      AQ(6,N1) = Q6
      AQ(7,N1) = Q7
      AQ(8,N1) = Q8
  100 CONTINUE
C
      DO 101  I1=1,8
      V(I1) = AQ(I1,N1)
  101 CONTINUE
C
      AT(N1) = T
      NNN = N1 + 2
      N1 = N1+1
C
C     ++++++++++++++++++++++++++++++++++++++++++++
C     +                                          +
C     +            MATRIX MULTIPLICATION         +
C     +                                          +
C     ++++++++++++++++++++++++++++++++++++++++++++
C
      DO 103 I3=1,8
      DUMMY = 0.
      DO 102 I2=1,8
      MATM = K(I3,I2)*V(I2)
      DUMMY = DUMMY + MATM
  102 CONTINUE
      DQ(I3,NNN) = DUMMY
      AQ(I3,N1) = DELT*(1.9167*DQ(I3,NNN)-1.3333*DQ(I3,NNN-1)
     &+0.4167*DQ(I3,NNN-2)) + V(I3)
  103 CONTINUE
C
      T = T+DELT
      IF (TMAX.GT.T) GO TO 100
      WRITE (6,1010)
 1010 FORMAT (//,5X,'T,Q1,Q2,Q3,Q4 ARE AS FOLLOWED:',/)
C
C        NOTE:  ONLY EVERY FIFTH VALUE IS PRINTED
C
      WRITE (6,1011) (AT(J),(AQ(IZ,J),IZ=1,4),J=1,N1,5)
 1011 FORMAT (' ',5E15.4)
C
C     ++++++++++++++++++++++++++++++++++++++++++++
C     +                                          +
C     +              PLOTTING FUNCTION           +
C     +                                          +
C     ++++++++++++++++++++++++++++++++++++++++++++
C
C
C        NOTE:  ONLY EVERY FIFTH VALUE IS PLOTTED
      N3 = 0
      DO 104  N2=1,N1,5
      N3 = N3 + 1
      TT(N3) = AT(N2)
      TQ1(N3)=AQ(1,N2)
      TQ2(N3) = AQ(2,N2)
      TQ3(N3) = AQ(3,N2)
```

```
              TQ4(N3) = AQ(4,N2)
      104     CONTINUE
      C
              WRITE (6,1012)
      1012    FORMAT ('1')
              CALL WPLOT1 (TT,TQ1,N3,12,'DISPLACEMENT')
              WRITE (6,1013)  TT(1),TMAX,DELT
      1013    FORMAT (40X,'DISPLACEMENT(1) AS A FUNCTION OF TAU',//,20X,
             1'T=',1F7.4,10X,'TMAX=',1F7.4,10X,'DELT=',1F7.4)
              WRITE (6,1012)
              CALL WPLOT1 (TT,TQ2,N3,12,'DISPLACEMENT')
              WRITE (6,1014)  TT(1),TMAX,DELT
      1014    FORMAT (40X,'DISPLACEMENT(2) AS A FUNCTION OF TAU',//,20X,
             1'T=',1F7.4,10X,'TMAX=',1F7.4,10X,'DELT=',1F7.4)
              WRITE (6,1012)
              CALL WPLOT1 (TT,TQ3,N3,12,'DISPLACEMENT')
              WRITE (6,1015)  TT(1),TMAX,DELT
      1015    FORMAT (40X,'DISPLACEMENT(3) AS A FUNCTION OF TAU',//,20X,
             1'T=',1F7.4,10X,'TMAX=',1F7.4,10X,'DELT=',1F7.4)
              WRITE (6,1012)
              CALL WPLOT1 (TT,TQ4,N3,12,'DISPLACEMENT')
              WRITE (6,1016)  TT(1),TMAX,DELT
      1016    FORMAT (40X,'DISPLACEMENT(4) AS A FUNCTION OF TAU',//,20X,
             1'T=',1F7.4,10X,'TMAX=',1F7.4,10X,'DELT=',1F7.4)
              WRITE (6,1012)
              GO TO 998
      999     CONTINUE
              STOP
              END

      $ENTRY
```

```
$JOB            0761536.SELFFD PIN=A82
C
C
C
C       ++++++++++++++++++++++++++++++++++++++++++++
C       +                                          +
C       +    NUMERICAL SOLUTION FOR STATIC AND     +
C       +   DYNAMIC INSTABILITIES OF A PROPELLANT  +
C       +       LINE.  (FOUR MODES ANALYSIS)       +
C       +      NB:  WITH FORMULA OF OPEN TYPES     +
C       +           NONLINEAR AND IMPROVED         +
C       +                                          +
C       ++++++++++++++++++++++++++++++++++++++++++++
C
        INTEGER  IT
        INTEGER  IA, IB, IC, ID1, ID2
        INTEGER  I1, I2, I3, I4, I5, I6
        INTEGER  JT
        INTEGER  J1, J2, J3, J4, J5, J6
        REAL  DUMMY, MATM
        REAL  RMU
        REAL  BETA, MU, NU, M, MF, I, E
        REAL  Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8
        REAL  LAM(4), VF
        REAL  L, Z1, T, TMAX, DELT
        REAL  DEL(4,4)
        REAL  DQ(8,3200)
        REAL  A(4,4), B(4,4), C(4,4)
        REAL  K(8,28), AQ(8,3200), AT(3200), V(28)
        REAL  TQ1(500), TQ2(500), TQ3(500), TQ4(500)
        REAL  TT(500)
C
C             Q1, Q2, Q3, ...  ARE INITIAL CONDITIONS
C             DQ(N)'S ARE PSEUDO INITIAL CONDITIONS
C             LAM(N) ARE MODE SHAPE PARAMETERS
C             VF = FLUID VELOCITY
C             L = LENGTH OF PIPE
C             Z1 = CRITICAL DAMPING RATIO
C             M = MASS OF PIPE
C             MF = MASS OF FLUID
C             I = INERTIA
C             E = MODULUS OF ELASTICITY
C             A(N), B(N), C(N) ARE MODE SHAPE INTEGRALS
C
        READ (5,1000)  Q1,Q2,Q3,Q4,Q5,Q6,Q7,Q8
 1000   FORMAT (8F10.4)
        READ (5,1005)  (LAM(I1), I1=1,4)
        READ (5,1005)  ((A(I1,J1), J1=1,4), I1=1,4)
        READ (5,1005)  ((B(I1,J1), J1=1,4), I1=1,4)
        READ (5,1005)  ((C(I1,J1), J1=1,4), I1=1,4)
 1005   FORMAT (4F10.4)
        READ (5,1010)  M,MF,I,E
 1010   FORMAT (4E10.3)
 998    READ (5,1011,END=999) VF,L,Z1,T,DELT,TMAX
 1011   FORMAT (F12.4,5F10.4)
C
C             BETA = DIMENSIONLESS FLUID VELOCITY
C             MU = MASS RATIO
C             NU = VISCOUS DAMPING COEFFICIENT
C
```

```
          BETA = VF*SQRT(MF*L**2./(E*I))
          MU = MF/(MF+M)
          NU = 7.04*Z1*SQRT(1.-MU)
C
          WRITE (6,1015)
 1015     FORMAT ('1')
          WRITE (6,1020)   ((A(I2,J2), J2=1,4), I2=1,4)
 1020     FORMAT (' ','A(I,J)=',4F20.4,/3(8X,4F20.4,/))
          WRITE (6,1025)   ((B(I2,J2), J2=1,4), I2=1,4)
 1025     FORMAT (' ','B(I,J)=',4F20.4,/3(8X,4F20.4,/))
          WRITE (6,1030)   ((C(I2,J2), J2=1,4), I2=1,4)
 1030     FORMAT (' ','C(I,J)=',4F20.4,/3(8X,4F20.4,/))
          WRITE (6,1035)   Q1,Q2,Q3,Q4,Q5,Q6,Q7,Q8
 1035     FORMAT (' ','Q(N)=',4F20.4,/,6X,4F20.4)
          WRITE (6,1040)   BETA,MU,NU
 1040     FORMAT (' ','BETA=',1F15.4,5X,'MU=',1F15.4,5X,'NU=',1F15.4)
          WRITE (6,1045)   VF,L,Z1,(IAM(II), II=1,4)
 1045     FORMAT (' ','VF=',1F15.4,5X,'L=',1F10.4,5X,'Z1=',1F10.4,/,' ',
         *'LAMDA(N)=',4F10.4)
          WRITE (6,1050)   M,MF,I,E
 1050     FORMAT (' ','M,MF,I,E ARE',4E20.3)
          WRITE (6,1055)
 1055     FORMAT (//,30X,'MATRIX IS')
          WRITE (6,1056)
 1056     FORMAT (' ','(DUE TO SPACE LIMITATION, MATRIX IS PRINTED AS (2?,
         * INSTEAD OF (8,28)',/)
C
          RMU = SQRT(MU)
C
C         ++++++++++++++++++++++++++++++++++++++++++++
C         +                                          +
C         +             ENTRIES OF MATRIX            +
C         +                                          +
C         ++++++++++++++++++++++++++++++++++++++++++++
C
C            NOTE:  DELTA FUNCTION
C
          DO 110  ID1=1,4
          DO 100  ID2=1,4
          DEL(ID1,ID2) = 0.0
  100     CONTINUE
          DEL(ID1,ID1) = 1.0
  110     CONTINUE
C
          DO 120  I4=1,8
          DQ(I4,1)  = 0.0
          DQ(I4,2)  = 0.0
  120     CONTINUE
C
C            NOTE:  DO-LOOP TO FILL IN 0'S & 1'S
C
          IC = 4
          DO 140  IA=1,4
          DO 130  IB=1,28
          K(IA,IB)  = 0.0
  130     CONTINUE
          IC = IC + 1
          K(IA,IC)  = 1.0
  140     CONTINUE
C
```

```
      C           NOTE:  ENTRIES WITH RECOGNIZED PATTERN
      C
                DO 160  I5=5,8
                IT = I5-4
                DO 150  J5=1,4
                K(I5,J5)  = -(DEL(IT,J5)*LAM(IT)**4.0+BETA**2.0*B(IT,J5))
        150     CONTINUE
                DO 155  J3=5,8
                JT= J3-4
                K(I5,J3)  = -(2.0*BETA*RMU*A(IT,JT)+DEL(IT,JT)*NU)
        155     CONTINUE
      C
                K(I5,9)  = B(IT,1)*C(1,1)
                K(I5,10)  = B(IT,1)*C(2,2)+B(IT,2)*(C(1,2)+C(2,2))
                K(I5,11)  = B(IT,1)*C(3,3)+B(IT,3)*(C(1,3)+C(3,1))
                K(I5,12)  = B(IT,1)*C(4,4)+B(IT,4)*(C(1,4)+C(4,1))
                K(I5,13)  = B(IT,1)*(C(1,2)+C(2,1))  + B(IT,2)*C(1,1)
                K(I5,14)  = B(IT,1)*(C(1,3)+C(3,1))+B(IT,3)*C(1,1)
                K(I5,15)  = B(IT,1)*(C(2,3)+C(3,2))+B(IT,2)*(C(1,3)+C(3,1))+
               #B(IT,3)*(C(1,2)+C(2,1))
                K(I5,16)  = B(IT,1)*(C(1,4)+C(4,1))+B(IT,4)*C(1,1)
                K(I5,17)  = B(IT,1)*(C(2,4)+C(4,2))+B(IT,2)*(C(1,4)+C(4,1))+
               #B(IT,4)*(C(1,2)+C(2,1))
                K(I5,18)  = B(IT,1)*(C(3,4)+C(4,3))+B(IT,3)*(C(1,4)+C(4,1))+
               #B(IT,4)*(C(1,3)+C(3,1))
                K(I5,19)  = B(IT,2)*C(2,2)
                K(I5,20)  = B(IT,2)*C(3,3)  + B(IT,3)*(C(2,3)+C(3,2))
                K(I5,21)  = B(IT,2)*(C(2,3)+C(3,2))+B(IT,3)*C(2,2)
                K(I5,22)  = B(IT,2)*C(4,4)+B(IT,4)*(C(2,4)+C(4,2))
                K(I5,23)  = B(IT,4)*C(2,2)+B(IT,2)*(C(2,4)+C(4,2))
                K(I5,24)  = B(IT,2)*(C(3,4)+C(4,3))+B(IT,3)*(C(2,4)+C(4,2))+
               #B(IT,4)*(C(2,3)+C(3,2))
                K(I5,25)  = B(IT,3)*C(3,3)
                K(I5,26)  = B(IT,3)*C(4,4)+B(IT,4)*(C(3,4)+C(4,3))
                K(I5,27)  = B(IT,4)*C(3,3)+B(IT,3)*(C(3,4)+C(4,3))
                K(I5,28)  = B(IT,4)*C(4,4)
      C
        160     CONTINUE
      C
      C
                WRITE (6,1060)   ((K(IA,JA), IA=1,8),JA=1,28)
        1060    FORMAT (' ',8E14.4)
      C
                N1 = 1
                AQ(1,N1)  = Q1
                AQ(2,N1)  = Q2
                AQ(3,N1)  = Q3
                AQ(4,N1)  = Q4
                AQ(5,N1)  = Q5
                AQ(6,N1)  = Q6
                AQ(7,N1)  = Q7
                AQ(8,N1)  = Q8
        170     CONTINUE
      C
                DO 180  IZ1=1,8
                V(IZ1) = AQ(IZ1,N1)
        180     CONTINUE
      C
      C
      C           NOTE:  HAVE TO ADD NON LINEAR EQUATIONS
```

```
C
      V(9) = AQ(1,N1)**3
      V(10) = AQ(1,N1)*AQ(2,N1)**2
      V(11) = AQ(1,N1)*AQ(3,N1)**2
      V(12) = AQ(1,N1)*AQ(4,N1)**2
      V(13) = AQ(1,N1)**2*AQ(2,N1)
      V(14) = AQ(1,N1)**2*AQ(3,N1)
      V(15) = AQ(1,N1)*AQ(2,N1)*AQ(3,N1)
      V(16) = AQ(1,N1)**2*AQ(4,N1)
      V(17) = AQ(1,N1)*AQ(2,N1)*AQ(4,N1)
      V(18) = AQ(1,N1)*AQ(3,N1)*AQ(4,N1)
      V(19) = AQ(2,N1)**3
      V(20) = AQ(2,N1)*AQ(3,N1)**2
      V(21) = AQ(2,N1)**2*AQ(3,N1)
      V(22) = AQ(2,N1)*AQ(4,N1)**2
      V(23) = AQ(2,N1)**2*AQ(4,N1)
      V(24) = AQ(2,N1)*AQ(3,N1)*AQ(4,N1)
      V(25) = AQ(3,N1)**3
      V(26) = AQ(3,N1)*AQ(4,N1)**2
      V(27) = AQ(3,N1)**2*AQ(4,N1)
      V(28) = AQ(4,N1)**3
C
C
      AT(N1) = T
      NNN = N1 + 2
      N1 = N1+1
C
C     ++++++++++++++++++++++++++++++++++++++++++++++
C     +                                            +
C     +           MATRIX MULTIPLICATION            +
C     +                                            +
C     ++++++++++++++++++++++++++++++++++++++++++++++
C
      DO 200  IZ2=1,8
      DUMMY = 0.
      DO 190  IZ3=1,28
      MATM = K(IZ2,IZ3)*V(IZ3)
      DUMMY = DUMMY + MATM
  190 CONTINUE
      DQ(IZ2,NNN) = DUMMY
      AQ(IZ2,N1) = DELT*(1.9167*DQ(IZ2,NNN)-1.3333*DQ(IZ2,NNN-1)+
     *0.4167*DQ(IZ2,NNN-2))+AQ(IZ2,N1-1)
  200 CONTINUE
C
      T = T+DELT
      IF (TMAX.GT.T) GO TO 170
C
      WRITE (6,1065)
 1065 FORMAT (//,5X,'T,Q1,Q2,Q3,Q4 ARE AS FOLLOWED:',/)
C
C        NOTE:  ONLY THE FIRST 200 VALUES AFTER 1.8 SEC ARE PRINTED
C
      WRITE (6,1070)  (AT(JP), (AQ(IP,JP), IP=1,4), JP=1000,1200,1)
 1070 FORMAT (' ',5E15.4)
C
C     ++++++++++++++++++++++++++++++++++++++++++++++
C     +                                            +
C     +             PLOTTING FUNCTION              +
C     +                                            +
C     ++++++++++++++++++++++++++++++++++++++++++++++
```

```
C
C      NOTE:  ONLY EVERY TENTH VALUE IS PLOTTED
C
       N3 = 0
       DO 210  N2=1,N1,10
       N3 = N3 + 1
       TT(N3) = AT(N2)
       TQ1(N3)=AQ(1,N2)
       TQ2(N3) = AQ(2,N2)
       TQ3(N3) = AQ(3,N2)
       TQ4(N3) = AQ(4,N2)
210    CONTINUE
C
       WRITE (6,1015)
       CALL WPLOT1 (TT,TQ1,N3,12,'DISPLACEMENT')
       WRITE (6,1071)  TT(1),TMAX,DELT
1071   FORMAT (40X,'DISPLACEMENT(1) AS A FUNCTION OF TAU',//,20X,
      1'T=',1F7.4,10X,'TMAX=',1F7.4,10X,'DELT=',1F7.4)
       WRITE (6,1015)
       CALL WPLOT1 (TT,TQ2,N3,12,'DISPLACEMENT')
       WRITE (6,1075)  TT(1),TMAX,DELT
1075   FORMAT (40X,'DISPLACEMENT(2) AS A FUNCTION OF TAU',//,20X,
      1'T=',1F7.4,10X,'TMAX=',1F7.4,10X,'DELT=',1F7.4)
       WRITE (6,1015)
       CALL WPLOT1 (TT,TQ3,N3,12,'DISPLACEMENT')
       WRITE (6,1080)  TT(1),TMAX,DELT
1080   FORMAT (40X,'DISPLACEMENT(3) AS A FUNCTION OF TAU',//,20X,
      1'T=',1F7.4,10X,'TMAX=',1F7.4,10X,'DELT=',1F7.4)
       WRITE (6,1015)
       CALL WPLOT1 (TT,TQ4,N3,12,'DISPLACEMENT')
       WRITE (6,1085)  TT(1),TMAX,DELT
1085   FORMAT (40X,'DISPLACEMENT(4) AS A FUNCTION OF TAU',//,20X,
      1'T=',1F7.4,10X,'TMAX=',1F7.4,10X,'DELT=',1F7.4)
       WRITE (6,1015)
       GO TO 998
999    CONTINUE
       STOP
       END

$ENTRY
```

APPENDIX V

Mode Shape Integrals of a Simply Supported Beam

$$\int_0^1 \gamma_j \, \gamma_k^{iv} \, d\xi = k^4 \pi^4 \int_0^1 \sin j\pi\xi \, \sin k\pi\xi \, d\xi \tag{A-21}$$

$$= \begin{cases} \dfrac{k^4\pi^4}{2} & j=k \\ 0 & j\neq k \end{cases}$$

$$= \frac{k^4\pi^4}{2} \, \delta_{jk}$$

$$\int_0^1 \gamma_j \, \gamma_k \, d\xi = \int_0^1 \sin j\pi\xi \, \sin k\pi\xi \, d\,\xi \tag{A-22}$$

$$= \begin{cases} \dfrac{1}{2} & j=k \\ 0 & j\neq k \end{cases}$$

$$= \frac{1}{2} \, \delta_{jk}$$

$$\int_0^1 \gamma_j \gamma_k' \, d\xi = k\pi \int_0^1 \sin j\pi\xi \, \cos k\pi\xi \, d\xi \tag{A-23}$$

$= a_{jk}$ (values of a_{jk} are tabulated in Table 4)

$$\int_0^1 \gamma_j \, \gamma_k'' \, d\xi = -k^2\pi^2 \int_0^1 \sin j\pi\xi \, \sin k\pi\xi \, d\xi \tag{A-24}$$

$$= \begin{cases} \dfrac{-k^2\pi^2}{2} & j=k \\ 0 & j\neq k \end{cases}$$

$$= \frac{-k^2\pi^2}{2} \, \delta_{jk}$$

$$= b_{jk}$$

$$\int_0^1 \gamma_h' \, \gamma_i' \, d\xi = hi \, \pi^2 \int_0^1 \cos h\pi\xi \, \cos i\pi\xi \, d\xi \tag{A-25}$$

$$= \begin{cases} \dfrac{h^2\pi^2}{2} & h=i \\ 0 & h\neq i \end{cases}$$

$$= \frac{-k^2\pi^2}{2} \, \delta_{hi}$$

$$= c_{hi}$$

APPENDIX VI

A LISTING OF FORTRAN IV PROGRAM FOR

SIMPLY SUPPORTED TUBE

```
             C
             C
             C      ++++++++++++++++++++++++++++++++++++++++++++
             C      +                                          +
             C      +                                          +
             C      +              DIVERGENCE                  +
             C      +          SIMPLY-SUPPORTED                +
             C      +          SPECIAL EDITION                 +
             C      +                                          +
             C      +                                          +
             C      ++++++++++++++++++++++++++++++++++++++++++++
             C
0001                REAL  A(4,4), B(4,4), C(4,4)
0002                REAL  DEL(4,4)
0003                REAL  MATM,MU,NU,M,MF,I
0004                REAL  LAM(4),L
0005                REAL  DQ(8,2500)
0006                REAL  K(8,28),AQ(8,2020),AT(2020),V(28)
0007                REAL  TQ1(2020),TT(2020),TQ2(2020)
             C
             C
0008                READ (5,1000)   (AQ(IU,1), IU=1,8)
0009         1000   FORMAT (8F10.4)
0010                READ (5,1005)   ((A(I1,J1), J1=1,4), I1=1,4)
0011         1005   FORMAT (4F10.4)
0012                READ (5,1010)   M,MF,I,E
0013         1010   FORMAT (4E10.3)
0014                READ (5,1011)   VF,L,Z1
0015         1011   FORMAT (F12.4,2F10.4)
0016                N3 = 0
             C
             C
0017                BETA = VF*SQRT(MF*L**2./(E*I))
0018                MU = MF/(MF+M)
0019                NU = 7.04*Z1*SQRT(1.-MU)
             C
0020                WRITE (6,1015)
0021         1015   FORMAT ('1')
             C
             C         NOTE:  DELTA FUNCTION
             C                B(J,K),C(K,I) ... ARE DIFFERENT
             C
0022                DO 110  ID1=1,4
0023                DO 100  ID2=1,4
0024                DEL(ID1,ID2) = 0.0
0025                B(ID1,ID2) = 0.0
0026                C(ID1,ID2) = 0.0
0027         100    CONTINUE
0028                DEL(ID1,ID1) = 0.5
0029                LAM(ID1) = 3.14159*ID1
0030                B(ID1,ID1) = -0.5*LAM(ID1)**2
0031                C(ID1,ID1) = 0.5*LAM(ID1)**2
0032         110    CONTINUE
             C
             C
0033                RMU = SQRT(MU)
```

```
              C
              C      +++++++++++++++++++++++++++++++++++++++++
              C      +                                       +
              C      +          ENTRIES OF MATRIX            +
              C      +                                       +
              C      +++++++++++++++++++++++++++++++++++++++++
              C
 0034                DO 120  I4=1,8
 0035                DQ(I4,1) = 0.0
 0036                DQ(I4,2) = 0.0
 0037         120    CONTINUE
              C
              C        NOTE:  DO-LOOP TO FILL IN 0'S & 1'S
              C
 0038                IC = 4
 0039                DO 140  IA=1,4
 0040                DO 130  IB=1,28
 0041                K(IA,IB) = 0.0
 0042         130    CONTINUE
 0043                IC = IC + 1
 0044                K(IA,IC) = 1.0
 0045         140    CONTINUE
              C
              C        NOTE:  ENTRIES WITH RECOGNIZED PATTERN
              C
 0046                DO 160  I5=5,8
 0047                IT = I5-4
 0048                DO 150  J5=1,4
 0049                K(I5,J5) = -(DEL(IT,J5)*LAM(IT)**4.0+BETA**2.0*B(IT,J5))
 0050         150    CONTINUE
 0051                DO 155  J3=5,8
 0052                JT= J3-4
 0053                K(I5,J3) = -(2.0*BETA*RMU*B(IT,JT)+DEL(IT,JT)*MU)
 0054         155    CONTINUE
              C
 0055                K(I5,9)  = B(IT,1)*C(1,1)
 0056                K(I5,10) = B(IT,1)*C(2,2)+B(IT,2)*(C(1,2)+C(2,2))
 0057                K(I5,11) = B(IT,1)*C(3,3)+B(IT,3)*(C(1,3)+C(3,1))
 0058                K(I5,12) = B(IT,1)*C(4,4)+B(IT,4)*(C(1,4)+C(4,1))
 0059                K(I5,13) = B(IT,1)*(C(1,2)+C(2,1))  + B(IT,2)*C(1,1)
 0060                K(I5,14) = B(IT,1)*(C(1,3)+C(3,1))+B(IT,3)*C(1,1)
 0061                K(I5,15) = B(IT,1)*(C(2,3)+C(3,2))+B(IT,2)*(C(1,3)+C(3,[illegible]
                    *B(IT,3)*(C(1,2)+C(2,1))
 0062                K(I5,16) = B(IT,1)*(C(1,4)+C(4,1))+B(IT,4)*C(1,1)
 0063                K(I5,17) = B(IT,1)*(C(2,4)+C(4,2))+B(IT,2)*(C(1,4)+C(4,[illegible]
                    *B(IT,4)*(C(1,2)+C(2,1))
 0064                K(I5,18) = B(IT,1)*(C(3,4)+C(4,3))+B(IT,3)*(C(1,4)+C(4,[illegible]
                    *B(IT,4)*(C(1,3)+C(3,1))
 0065                K(I5,19) = B(IT,2)*C(2,2)
 0066                K(I5,20) = B(IT,2)*C(3,3)  + B(IT,3)*(C(2,3)+C(3,2))
 0067                K(I5,21) = B(IT,2)*(C(2,3)+C(3,2))+B(IT,3)*C(2,2)
 0068                K(I5,22) = B(IT,2)*C(4,4)+B(IT,4)*(C(2,4)+C(4,2))
 0069                K(I5,23) = B(IT,4)*C(2,2)+B(IT,2)*(C(2,4)+C(4,2))
 0070                K(I5,24) = B(IT,2)*(C(3,4)+C(4,3))+B(IT,3)*(C(2,4)+C(4,[illegible]
                    *B(IT,4)*(C(2,3)+C(3,2))
 0071                K(I5,25) = B(IT,3)*C(3,3)
```

```
0072            K(I5,26)  = B(IT,3)*C(4,4)+B(IT,4)*(C(3,4)+C(4,3))
0073            K(I5,27)  = B(IT,4)*C(3,3)+B(IT,3)*(C(3,4)+C(4,3))
0074            K(I5,28)  = B(IT,4)*C(4,4)
          C
0075      160   CONTINUE
          C
          C        2 CONTINUATION CARDS JUST IN   SE
0076      165   CONTINUE
          C
0077            READ (5,1012,END=999)  T,DELT,TMA
0078      1012  FORMAT (3F10.4)
0079            WRITE (6,5554)  T,DELT,TMAX
0080      5554  FORMAT( '    ',3F20.4)
0081            N1 = 1
          C
0082      170   CONTINUE
          C
0083            DO 180  IZ1=1,8
0084            V(IZ1)  =  AQ(IZ1,N1)
0085      180   CONTINUE
          C
          C
          C        NOTE:  HAVE TO ADD NON LINEAR EQUATIONS
          C
0086            V(9)  =  AQ(1,N1)**3
0087            V(10)  =  AQ(1,N1)*AQ(2,N1)**2
0088            V(11)  =  AQ(1,N1)*AQ(3,N1)**2
0089            V(12)  =  AQ(1,N1)*AQ(4,N1)**2
0090            V(13)  =  AQ(1,N1)**2*AQ(2,N1)
0091            V(14)  =  AQ(1,N1)**2*AQ(3,N1)
0092            V(15)  =  AQ(1,N1)*AQ(2,N1)*AQ(3,N1)
0093            V(16)  =  AQ(1,N1)**2*AQ(4,N1)
0094            V(17)  =  AQ(1,N1)*AQ(2,N1)*AQ(4,N1)
0095            V(18)  =  AQ(1,N1)*AQ(3,N1)*AQ(4,N1)
0096            V(19)  =  AQ(2,N1)**3
0097            V(20)  =  AQ(2,N1)*AQ(3,N1)**2
0098            V(21)  =  AQ(2,N1)**2*AQ(3,N1)
0099            V(22)  =  AQ(2,N1)*AQ(4,N1)**2
0100            V(23)  =  AQ(2,N1)**2*AQ(4,N1)
0101            V(24)  =  AQ(2,N1)*AQ(3,N1)*AQ(4,N1)
0102            V(25)  =  AQ(3,N1)**3
0103            V(26)  =  AQ(3,N1)*AQ(4,N1)**2
0104            V(27)  =  AQ(3,N1)**2*AQ(4,N1)
0105            V(28)  =  AQ(4,N1)**3
          C
          C
0106            AT(N1)  =  T
0107            NNN = N1 + 2
0108            N1  =  N1+1
          C
          C     ++++++++++++++++++++++++++++++++++++++++++++++++
          C     +                                              +
          C     +             MATRIX MULTIPLICATION            +
          C     +                                              +
          C     ++++++++++++++++++++++++++++++++++++++++++++++++
          C
```

```
              DO 200  IZ2=1,8
              DUMMY = 0.
              DO 190  IZ3=1,28
              MATM = K(IZ2,IZ3)*V(IZ3)
              DUMMY = DUMMY + MATM
      190     CONTINUE
              DQ(IZ2,NNN) = DUMMY
              AQ(IZ2,N1) = DELT*(1.9167*DQ(IZ2,NNN)-1.3333*DQ(IZ2,NNN-1)
             *0.4167*DQ(IZ2,NNN-2))+AQ(IZ2,N1-1)
      200     CONTINUE
      C
              T = T+DELT
              IF (TMAX.GT.T) GO TO 170
      C
      C
              DO 210  N2=1,N1,10
              N3 = N3+1
              TT(N3) = AT(N2)
              TQ1(N3) = AQ(1,N2)
              TQ2(N3) = AQ(2,N2)
      210     CONTINUE
      C
              DO 220  IS=1,8
              DQ(IS,2) = DQ(IS,NNN-1)
              DQ(IS,1) = DQ(IS,NNN-2)
              AQ(IS,1) = AQ(IS,N1)
      220     CONTINUE
              N3 = N3-1
              GO TO 165
      C
      999     CONTINUE
      C
      C
      C       ++++++++++++++++++++++++++++++++++++++++++++
      C       +                                          +
      C       +             PLOTTING FUNCTION            +
      C       +                                          +
      C       ++++++++++++++++++++++++++++++++++++++++++++
      C
      C          NOTE:  ONLY EVERY TENTH VALUE IS PLOTTED
      C
      C
              WRITE (6,5555) N3,TT(1),TT(N3)
      5555    FORMAT ('      ',I6,2F10.4)
              CALL DFIPS1(TT,TQ1,N3,1,10.0)
              CALL DFIPS1(TT,TQ2,N3,1,10.0)
              WRITE (6,1015)
              STOP
              END
```

END

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